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A Graded Soft Set Framework for Multi-Criteria Decision-Making Under Uncertainty: Weighted Ranking and Parameter Reduction

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Abstract

Decision-making under uncertainty is often complex and involves incomplete and imprecise information on several criteria. To address these challenges, this study introduces a graded soft set framework by incorporating membership and complementary inverse membership functions into soft set theory. Basic set operations and their algebraic properties are set up. Three algorithms are developed to solve the alternative selection problem, the weighted ranking problem and the parameter reduction problem. The proposed approaches use criterion weights and the grading scores for the evaluation of the alternatives and the determination of the less significant parameters. Applications to supplier selection and symptom-based assessment illustrate the framework's practical utility. The suggested methodology provides a straightforward and interpretable decision-supporting tool for multi-criteria decision-making, which needs to be validated in sensitivity analysis and compared to other existing methodologies.

Keywords: Graded Soft Sets, Multi-Criteria Decision-Making (MCDM), Uncertainty Modelling, Membership Functions, Weighted Ranking Algorithm, Parameter Reduction, Supplier Selection, Decision Analysis

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1. Introduction

Decision-making under uncertainty is a fundamental problem in mathematics, computer science, engineering, management, and other applied disciplines. In many practical situations, decision-makers must evaluate a finite collection of alternatives against multiple criteria when the available information is incomplete, imprecise, or expressed through subjective judgments. Traditional decision models may require probability distributions, precise numerical measurements, or predefined membership functions, which are not always readily available. Consequently, mathematical frameworks that represent uncertainty flexibly and interpretably remain important for developing reliable decision-support methodologies.

Zadeh (1965) introduced fuzzy set theory to represent uncertainty through graded membership values, providing a foundation for subsequent developments in uncertainty modelling. Pawlak (1982) proposed rough set theory, which uses lower and upper approximations to analyse imprecise information. Atanassov (1986) extended fuzzy set theory by introducing intuitionistic fuzzy sets, incorporating both membership and non-membership degrees. Molodtsov (1999) introduced soft set theory as a parameterised mathematical framework for handling uncertainty without requiring predefined membership functions. Maji et al. (2001) developed fuzzy soft sets by integrating fuzzy membership with the parameterised structure of soft sets. In the same year, Maji et al. (2001) introduced intuitionistic fuzzy soft sets to represent membership and non-membership information in uncertain environments. Maji et al. (2002) demonstrated the practical application of soft set theory to decision-making problems, while Maji et al. (2003) formalised its fundamental concepts and operations.

Kong et al. (2008) developed an algorithm for normal parameter reduction to identify essential parameters in soft set theory. Feng et al. (2010) proposed an adjustable approach to fuzzy soft set-based decision-making, enhancing flexibility in evaluating alternatives. Chen et al. (2005) investigated parameterisation reduction in soft sets and demonstrated its relevance to practical applications. More recently, Guo et al. (2023) introduced a parameter-reduction method for fuzzy soft sets to improve decision-making efficiency. Akram et al. (2023) proposed a multi-attribute group decision-making method based on hesitant fuzzy soft expert information. Ma et al. (2023) developed a K-nearest-neighbour-based method for handling incomplete interval-valued fuzzy soft set data. Malik et al. (2023) investigated medical decision-making techniques using bipolar soft information to represent uncertain assessments.

Alcantud et al. (2024) systematically reviewed soft set theory, highlighting its theoretical developments, extensions, and applications in decision-making. Priyadarsini et al. (2024) presented a neutrosophic soft set approach incorporating parameter reduction and graphical representation. Unni et al. (2024) developed a circular intuitionistic fuzzy soft set framework for decision-making problems. Zukri et al. (2024) examined parameter reduction in soft sets to simplify parameterised decision analysis. Alcantud et al. (2025) proposed scored-energy-based methods using hesitant fuzzy soft sets for decision-making and clustering. Dalkılıç (2025) developed a soft set-based decision-making approach emphasising the selection of the best parameter–object pair. Patil et al. (2025) proposed a ranking method for trapezoidal fuzzy soft sets and demonstrated its application to multi-criteria decision-making. Umul (2025) introduced an intuitionistic fuzzy soft set-based decision-making method that prioritises user rankings. Hazaymeh and Bataihah (2025) explored multiple opinions within fuzzy soft expert sets and their application to decision-making problems. Shahzadi et al. (2026) developed a multi-criteria decision-making model using circular Pythagorean fuzzy soft sets.

The proposed framework is designed to support decision problems in which alternatives are evaluated using normalised graded assessments and, where appropriate, criterion weights. The weighted ranking procedure aggregates these assessments into net scores that can be used to order the available alternatives.

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The parameter-reduction procedure seeks to identify criteria that contribute limited information or exhibit similar evaluation patterns. However, the removal of a criterion does not automatically guarantee preservation of the ranking; this property requires explicit mathematical conditions and empirical verification. The framework is accordingly explored in terms of definitions, algebraic properties, illustrative decision problems, and sensitivity considerations.

2. Preliminaries

Definition 2.1 (Graded Soft Set): Let U be a finite universe of objects, E be a set of parameters, and R be a relation between U and E. A graded soft set over U is a pair (A), where $A \subseteq E$and $F: A \to G(U)$is a mapping that assigns to each parameter $e \in A$a set of pairs $\left(\mu_F(e)(x)\right)$, where $x \in U$and $\mu_F(e)(x) \in [1]$represents the degree to which $x$satisfies parameter e.

The fundamental distinction between graded soft sets and classical soft sets lies in the replacement of the crisp approximation function. $F: A \to P(U)$with a graded version that assigns membership degrees to each element-parameter pair. This modification enables the representation of partial or graded satisfaction of parameters, capturing the nuanced uncertainty that characterises many decision-making scenarios.

Definition 2.2 (Graded Membership Function): For a graded soft set (A)over U, the graded membership function for parameter $e \in A$is defined as:

$$\mu_F(e): U \to [1]$$

where $\mu_F(e)(x)$represents the degree to which object $x$satisfies parameter e.

Definition 2.3 (Inverse Graded Membership Function): The inverse graded membership function for parameter $e \in A$with respect to object $x \in U$is defined as:

$$\nu_F(e)(x) = 1 - \mu_F(e)(x)$$

This function represents the degree to which object. $x$does not satisfy parameter e, providing a complementary perspective on the element-parameter relationship.

The introduction of both graded and inverse graded membership functions enables a more comprehensive characterisation of uncertainty. In decision-making contexts, the graded membership function captures the degree of satisfaction or compatibility, while the inverse graded membership function captures the degree of dissatisfaction or incompatibility. Together, these functions provide a dual perspective that is essential for nuanced decision analysis.

Definition 2.4 Properties of Graded Soft Sets: The proposed framework exhibits several important properties that distinguish it from classical soft sets and other extensions.

Property 2.1 (Granularity Preservation). For any graded soft set (A), if $\mu_F(e)(x) \in \{1\}$for all $e \in A$and $x \in U$, then (A)reduces to a classical soft set. This property demonstrates that the proposed framework is a proper generalisation of classical soft set theory.

Proof: If $\mu_F(e)(x) \in \{1\}$for all $e \in A$and $x \in U$, then for each parameter e, the set $\left\{x \in U: \mu_F(e)(x) = 1\right\}$ defines a subset of U. This subset corresponds exactly to $F(e)$in the classical soft set (A). Therefore, the graded soft set reduces to a classical soft set.

Property 2.2 (Parameter-Wise Decomposition). Each parameter $e \in A$induces a fuzzy set $F_e$on U, where the membership function of $F_e$is given by $\mu_F(e)$. This property enables the decomposition of a graded soft

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set into a collection of fuzzy sets, facilitating analysis through fuzzy set-theoretic tools.

Property 2.3 (Complementarity). For each parameter $e \in A$and object $x \in U$, the graded and inverse graded membership functions satisfy the complementarity condition:

$$\mu_F(e)(x) + \nu_F(e)(x) = 1$$

This property ensures consistency in the dual representation of element-parameter relationships.

Property 2.4 (Monotonicity). If $\mu_F(e)(x) \geq \mu_F\left(e^{'}\right)(x)$ for all $x \in U$, then the parameter $e$provides at least as much discriminatory information as parameter $e^{'}$with respect to the objects in U. This property has important implications for parameter reduction and feature selection.

Proof: If $\mu_F(e)(x) \geq \mu_F\left(e^{'}\right)(x)$for all $x \in U$, then for any threshold $\alpha \in [1]$, the set $\left\{x: \mu_F(e)(x) \geq \alpha\right\}$contains the set $\left\{x: \mu_F\left(e^{'}\right)(x) \geq \alpha\right\}$. Therefore, parameter $e$distinguishes at least as many objects as parameter $e^{'}$at every threshold level, providing greater discriminatory information.

Definition 2.5 Operations on Graded Soft Sets: We define fundamental operations on graded soft sets that parallel those of classical soft set theory while accommodating the graded membership structure.

Definition 2.6 (Union). The union of two graded soft sets $(A)$and (B)over $U$is defined as (C), where $C = A \cup B$and for all $e \in C$:

$$\mu_H(e)(x) = max\left((e)(x)\mu_G(e)(x)\right) \; if \; e \in A \cap B \mu_H(e)(x) = \mu_F(e)(x) \; if \; e \in A \backslash B \mu_H(e)(x) = \mu_G(e)(x) \; if \; e \in B \backslash A$$

Definition 2.7 (Intersection). The intersection of two graded soft sets $(A)$and (B)over $U$is defined as (C), where $C = A \cap B$and for all $e \in C$:

$$\mu_H(e)(x) = min\left((e)(x)\mu_G(e)(x)\right)$$

Definition 2.8 (Complement). The complement of a graded soft set (A)over $U$is defined as (A), where for all $e \in A$and $x \in U$:

$$\mu_{F^c}(e)(x) = 1 - \mu_F(e)(x) = \nu_F(e)(x)$$

Definition 2.9 (AND Operation). If $(A)$and $(B)$are two graded soft sets over U, then $(A) \wedge (B)$is defined as $(A \times B)$, where:

$$H(\beta)(x) = min((\alpha)(x)G(\beta)(x))$$

for all $(\beta) \in A \times B$and $x \in U$.

These operations preserve the graded membership structure and satisfy fundamental algebraic properties including commutativity, associativity, and distributivity.

Theorem 2.1 (De Morgan's Laws). For any two graded soft sets $(A)$and (B)over U:

$$((A) \cup (B))^c = (A)^c \cap (B)^c ((A) \cap (B))^c = (A)^c \cup (B)^c$$

Proof: For the first law, consider any $e \in A \cup B$. If $e \in A \cap B$, then:

$$\mu_{((A) \cup (B))^c}(e)(x) = 1 - max\left((e)(x)\mu_G(e)(x)\right)$$

By De Morgan's laws for real numbers:

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$$1 - max(b) = min(-a1 - b)$$

Therefore,

$$\mu_{((A)\cup(B))^c}(e)(x) = min\left(-\mu_F(e)(x)1 - \mu_G(e)(x)\right) = min\left((e)(x)\nu_G(e)(x)\right)$$

which equals $\mu_{(A)^c\cap(B)^c}(e)(x)$. The cases for $e\in A\backslash B$and $e\in B\backslash A$follow similarly. The second law follows by duality.

3. Decision-Making Algorithms

The proposed Graded Soft Set Theory framework enables the development of decision-making algorithms that leverage graded membership information for more nuanced decision analysis. We present three algorithms: the Graded Choice Value Algorithm, the Graded Weighted Ranking Algorithm, and the Graded Parameter Reduction Algorithm.

3.1 Graded Choice Value Algorithm: The Graded Choice Value Algorithm extends the classical choice value approach by incorporating graded membership degrees into the computation of decision scores.

Algorithm 3.1: Graded Choice Value Algorithm

text

Input Specification

Let the following be given:

Universe of Alternatives:

$$U = \{u_n\},\ n\in N$$

Set of Parameters:

$$E = \{e_m\},\ m\in N$$

Graded Soft Decision Matrix:

$$M = [m_{ij}]_{n\times m},\ m_{ij} \in [1]$$

where $m_{ij} = \mu_F(e_j)(u_i)$represents the graded membership degree of alternative $u_i$with respect to parameter e_j, and $\mu_F(e_j)$: $U\rightarrow$[1]is the graded membership function for parameter. e_j.

Parameter Weights:

$$W = (w_m),\ w_j \in [1],\ \sum_{j=1}^{m} w_j = 1$$

Output: A ranking of alternatives $R = (u_{\sigma(n)})$such that $s_{\sigma(1)} \geq s_{\sigma(2)}\geq\cdots\geq s_{\sigma(n)}$.

Step-by-Step Mathematical Procedure

Step 1: Construct the Graded Soft Decision Matrix

For each alternative $u_i\in U$ and each parameter $e_j\in E$, assign the graded membership value:

$$m_{ij} = \mu_F(e_j)(u_i),\ i = 1, 2, ..., n;\ j = 1, 2, ..., m$$

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The resulting matrix is:

$$M = \begin{pmatrix} m_{11} & m_{12} & \cdots & m_{1m} \\ m_{21} & m_{22} & \cdots & m_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n1} & m_{n2} & \cdots & m_{nm} \end{pmatrix}_{n \times m}$$

where each entry satisfies $0 \leq m_{ij} \leq 1$.

Step 2: Compute Graded Choice Values

For each alternative u_i, the graded choice value c_i is defined as the sum of its graded membership degrees across all parameters:

$$c_i = \sum_{j=1}^{m} m_{ij}, \; i = 1, 2, \ldots, n$$

Expanded form:

$$c_i = m_{i1} + m_{i2} + \cdots + m_{im}$$

Bounds:

$$0 \leq c_i \leq m, \; \forall i = 1, 2, \ldots, n$$

where $c_i = 0$ if $m_{ij} = 0$ for all j, and $c_i = m$ if $m_{ij} = 1$ for all j.

Step 3: Compute Inverse Graded Choice Values

For each alternative u_i, the inverse graded choice value d_i is defined as the sum of the complement degrees across all parameters:

$$d_i = \sum_{j=1}^{m} (1 - m_{ij}), \; i = 1, 2, \ldots, n$$

Expanded form:

$$d_i = \left(1 - m_{i1}\right) + \left(1 - m_{i2}\right) + \cdots + \left(1 - m_{im}\right)$$

Simplification:

$$d_i = m - \sum_{j=1}^{m} m_{ij} = m - c_i$$

Bounds:

$$0 \leq d_i \leq m, \; \forall i = 1, 2, \ldots, n$$

Step 4: Compute Net Graded Choice Values

For each alternative u_i, the net graded choice value s_i is defined as the difference between the graded choice value and the inverse graded choice value:

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$$s_i = c_i - d_i,\ i = 1, 2, ..., n$$

Substituting $d_i = m - c_i$**:**

$$s_i = c_i - \left(m - c_i\right) = 2c_i - m$$

Expanded form:

$$s_i = \sum_{j=1}^{m} m_{ij} - \sum_{j=1}^{m} (1 - m_{ij}) = \sum_{j=1}^{m} (m_{ij} - \left(1 - m_{ij}\right)) = \sum_{j=1}^{m} (2m_{ij} - 1)$$

Bounds:

$$-m \leq s_i \leq m,\ \forall i = 1, 2, ..., n$$

where:

$s_i = m$if and only if $m_{ij} = 1$for all $j = 1, 2, ..., m$

$s_i = -m$if and only if $m_{ij} = 0$for all $j = 1, 2, ..., m$

$s_i = 0$if and only if $c_i = m/2$

Step 5: Rank Alternatives

Define a permutation σ: $\{n\} \rightarrow \{n\}$such that:

$$s_{\sigma(1)} \geq s_{\sigma(2)} \geq \cdots \geq s_{\sigma(n)}$$

The ranking of alternatives is then given by:

$$\mathscr{R} = \left(u_{\sigma(1)}, u_{\sigma(2)}, \ldots, u_{\sigma(n)}\right)$$

where $u_{\sigma(1)}$is the optimal alternative.

Tie-Breaking Rule: If $s_{\sigma(k)} = s_{\sigma(k+1)}$for some k, the tie may be resolved by:

Comparing graded choice values: $c_{\sigma(k)}$vs. $c_{\sigma(k+1)}$(prefer higher c_i)

Comparing inverse graded choice values: $d_{\sigma(k)}$vs. $d_{\sigma(k+1)}$(prefer lower d_i)

Lexicographic ordering of alternative indices

Step 6: Return Ranking

Return the ordered list:

$$\mathscr{R} = \left(u_{\sigma(1)}, u_{\sigma(2)}, \ldots, u_{\sigma(n)}\right)$$

with corresponding net graded choice values:

$$\left(s_{\sigma(1)}, s_{\sigma(2)}, \ldots, s_{\sigma(n)}\right)$$

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Theorem 3.1 (Choice Value Bounds). For any alternative u_i in a graded soft set with m parameters, the net graded choice value s_i satisfies:

$$-m \leq s_i \leq m$$

with $s_i = m$ if and only if $\mu_F(e_j)(u_i) = 1$ for all j, and $s_i = -m$ if and only if $\mu_F(e_j)(u_i) = 0$ for all j.

Proof: The graded choice value $c_i = \sum_{j=1}^{m} m_{ij}$ satisfies $0 \leq c_i \leq m$. The inverse graded choice value $d_i = \sum_{j=1}^{m} (1 - m_{ij}) = m - c_i$ satisfies $0 \leq d_i \leq m$. Therefore, $s_i = c_i - d_i = 2c_i - m$ satisfies $-m \leq s_i \leq m$. The extreme cases follow directly from the definitions.

Algorithm 3.2 Graded Weighted Ranking Algorithm: In many decision-making scenarios, parameters are not equally important. The Graded Weighted Ranking Algorithm incorporates parameter weights to reflect their relative significance.

Input Specification

Let the following be given:

Universe of Alternatives:

$$U = \{u_n\},\ n \in N$$

Set of Parameters:

$$E = \{e_m\},\ m \in N$$

Graded Soft Decision Matrix:

$$M = [m_{ij}]_{n \times m},\ m_{ij} \in [1]$$

where $m_{ij} = \mu_F(e_j)(u_i)$ represents the graded membership degree of alternative u_i with respect to parameter e_j, and $\mu_F(e_j)$: $U \rightarrow [1]$ is the graded membership function for parameter? e_j.

Weight Vector:

$$W = (w_m),\ w_j \in [1],\ \sum_{j=1}^{m} w_j = 1$$

Output: A weighted ranking of alternatives $\mathcal{R} = (u_{\sigma(1)}, u_{\sigma(2)}, \ldots, u_{\sigma(n)})$ such that $t_{\sigma(1)} \geq t_{\sigma(2)} \geq \cdots \geq t_{\sigma(n)}$.

Step-by-Step Mathematical Procedure

Step 1: Construct the Graded Soft Decision Matrix

For each alternative $u_i \in U$ and each parameter $e_j \in E$, assign the graded membership value:

$$m_{ij} = \mu_F(e_j)(u_i),\ i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, m$$

The resulting matrix is:

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$$M = \begin{pmatrix} m_{11} & m_{12} & \cdots & m_{1m} \\ m_{21} & m_{22} & \cdots & m_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n1} & m_{n2} & \cdots & m_{nm} \end{pmatrix}_{n \times m}$$

where each entry satisfies $0 \leq m_{ij} \leq 1$.

Step 2: Compute Weighted Graded Scores

For each alternative u_i, the weighted graded score r_i is defined as the weighted sum of its graded membership degrees across all parameters:

$$r_i = \sum_{j=1}^{m} w_j \cdot m_{ij}, \; i = 1, 2, \ldots, n$$

Expanded form:

$$r_i = w_1 m_{i1} + w_2 m_{i2} + \cdots + w_m m_{im}$$

Bounds:

Since $0 \leq m_{ij} \leq 1$ and $w_j \geq 0$ with $\sum_{j=1}^{m} w_j = 1$:

$$0 \leq r_i \leq 1, \; \forall i = 1, 2, \ldots, n$$

where:

$r_i = 0$ if and only if $m_{ij} = 0$ for all j with $w_j > 0$

$r_i = 1$ if and only if $m_{ij} = 1$ for all j with $w_j > 0$

Step 3: Compute Weighted Inverse Graded Scores

For each alternative u_i, the weighted inverse graded score q_i is defined as the weighted sum of the complement degrees across all parameters:

$$q_i = \sum_{j=1}^{m} w_j \cdot \left(1 - m_{ij}\right), \; i = 1, 2, \ldots, n$$

Expanded form:

$$q_i = w_1\left(1 - m_{i1}\right) + w_2\left(1 - m_{i2}\right) + \cdots + w_m\left(1 - m_{im}\right)$$

Simplification:

$$q_i = \sum_{j=1}^{m} w_j - \sum_{j=1}^{m} w_j \cdot m_{ij} = 1 - r_i$$

since $\sum_{j=1}^{m} w_j = 1$.

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Bounds:

$$0 \leq q_i \leq 1, \ \forall i = 1, 2, \ldots, n$$

with $q_i = 1 - r_i$, confirming the complementarity relationship.

Step 4: Compute Weighted Net Scores

For each alternative u_i, the weighted net score t_i is defined as the difference between the weighted graded score and the weighted inverse graded score:

$$t_i = r_i - q_i, \ i = 1, 2, \ldots, n$$

Substituting $q_i = 1 - r_i$:

$$t_i = r_i - \left(1 - r_i\right) = 2r_i - 1$$

Expanded form:

$$t_i = \sum_{j=1}^{m} w_j \cdot m_{ij} - \sum_{j=1}^{m} w_j \cdot \left(1 - m_{ij}\right) = \sum_{j=1}^{m} w_j \cdot \left(m_{ij} - \left(1 - m_{ij}\right)\right) = \sum_{j=1}^{m} w_j \cdot \left(2m_{ij} - 1\right)$$

Bounds:

$$-1 \leq t_i \leq 1, \ \forall i = 1, 2, \ldots, n$$

where:

- $t_i = 1$ if and only if $m_{ij} = 1$ for all j with $w_j > 0$
- $t_i = -1$ if and only if $m_{ij} = 0$ for all j with $w_j > 0$
- $t_i = 0$ if and only if $r_i = 1/2$

Interpretation:

- $t_i > 0$: The alternative u_i has more positive evidence than negative evidence.
- $t_i = 0$: The alternative u_i has balanced positive and negative evidence.
- $t_i < 0$: The alternative u_i has more negative evidence than positive evidence.

Step 5: Rank Alternatives

Define a permutation $\sigma: \{n\} \to \{n\}$ such that:

$$t_{\sigma(1)} \geq t_{\sigma(2)} \geq \cdots \geq t_{\sigma(n)}$$

The weighted ranking of alternatives is then given by:

$$\mathcal{R} = \left(u_{\sigma(1)}, u_{\sigma(2)}, \ldots, u_{\sigma(n)}\right)$$

where $u_{\sigma(1)}$ is the optimal alternative under the weighted evaluation.

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Tie-Breaking Rules:

If $t_{\sigma(k)} = t_{\sigma(k+1)}$ for some k, the tie may be resolved by applying the following rules in sequence:

Weighted Graded Score Comparison: Prefer the alternative with higher r_i:

$$if\ t_i = t_j\ and\ r_i > r_j,\ then\ rank\ u_i\ above\ u_j$$

Weighted Inverse Graded Score Comparison: Prefer the alternative with lower q_i:

$$if\ t_i = t_j\ and\ q_i < q_j,\ then\ rank\ u_i\ above\ u_j$$

Parameter-Wise Comparison: Compare the graded membership values lexicographically according to parameter weights:

$$if\ t_i = t_j,\ compare\ \left(m_{im}\right)\ vs.\ \left(m_{jm}\right)$$

Index Ordering: If all else fails, order by alternative index:

$$if\ t_i = t_j\ and\ i < j,\ then\ rank\ u_i\ above\ u_j$$

Step 6: Return Weighted Ranking

Return the ordered list:

$$\mathcal{R} = \left(u_{\sigma(1)}, u_{\sigma(2)}, \ldots, u_{\sigma(n)}\right)$$

with corresponding weighted net scores:

$$\left(t_{\sigma(1)}, t_{\sigma(2)}, \ldots, t_{\sigma(n)}\right)$$

3.3 Graded Parameter Reduction Algorithm: The proposed framework also supports parameter reduction, which is essential for efficient decision-making when the parameter set is large or contains redundant information.

Definition 3.8 (Graded Parameter Significance). The significance of parameter $e \in A$ with respect to a graded soft set (A) is defined as:

$$\sigma(e) = \frac{1}{n} \sum_{x \in U} \mu_F(e)(x)$$

This measure captures the average degree to which objects satisfy the parameter. e.

Definition 3.9 (Graded Parameter Redundancy). Two parameters $e_i, e_j \in A$ are graded redundant if:

$$\sum_{x \in U} |\mu_F\left(e_i\right)(x) - \mu_F\left(e_j\right)(x)| < \varepsilon$$

for some threshold $\varepsilon > 0$.

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Algorithm 3.3: Graded Parameter Reduction Algorithm

Input Specification

Let the following be given:

Graded Soft Set:

$$(F, A) \text{ over universe } U$$

where:

- $U = \{x_n\}$is a finite universe of objects, $n = |U| \in N$
- $A = \{e_m\} \subseteq E$is a finite set of parameters, $m = |A| \in N$
- $F: A \to G(U)$is a mapping that assigns to each parameter $e \in A$a graded subset of U
- $\mu_F(e): U \to [1]$is the graded membership function for parameter? $e \in A$

Threshold Parameters:

$$\varepsilon \in [0, n],\ \delta \in [0, 1]$$

where:

ε is the redundancy threshold (maximum allowable distance for two parameters to be considered redundant)

δ is the significance threshold (minimum acceptable average membership value)

Output: A reduced parameter set $A^* \subseteq A$such that $A^* = A \backslash \left(A_{low} \cup A_{redundant}\right)$.

Step-by-Step Mathematical Procedure

Step 1: Compute Significance for Each Parameter

For each parameter $e \in A$, the significance $\sigma(e)$is defined as the average graded membership value across all objects in the universe:

$$\sigma(e) = \frac{1}{|U|} \sum_{x \in U} \mu_F(e)(x),\ \forall e \in A$$

Expanded form:

$$\sigma(e) = \frac{1}{n} \sum_{i=1}^{n} \mu_F(e)\left(x_i\right),\ \forall e \in A$$

Bounds:

Since $0 \leq \mu_F(e)(x) \leq 1$for all $e \in A$and $x \in U$:

$$0 \leq \sigma(e) \leq 1,\ \forall e \in A$$

where:

$\sigma(e) = 0$if and only if $\mu_F(e)(x) = 0$for all $x \in U$

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$\sigma(e) = 1$ if and only if $\mu_F(e)(x) = 1$ for all $x \in U$

Interpretation:

The significance measure $\sigma(e)$ quantifies the overall discriminatory power of parameter e. Parameters with low significance contribute little information to the decision-making process and may be candidates for removal.

Vector Representation:

Define the significance vector:

$$\sigma = \left(\sigma, (e_1), \sigma, (e_2) \ldots \sigma(e_m)\right)^T$$

This can be computed in matrix form as:

$$\sigma = \frac{1}{n} M^T \cdot 1_n$$

where:

$M = \left[\mu_F(e_j)(x_i)\right]_{n \times m}$ is the graded decision matrix

$1_n = (1)^T$ is the n-dimensional all-ones column vector

Step 2: Identify Low-Significance Parameters

The set of low-significance parameters A_{low} is defined as:

$$A_{low} = \{e \in A : \sigma(e) < \delta\}$$

Expanded form:

$$A_{low} = \left\{e \in A : \frac{1}{n} \sum_{i=1}^{n} \mu_F(e)(x_i) < \delta\right\}$$

Interpretation:

Parameters whose average membership value falls below the threshold δ are considered to provide insufficient discriminatory information and are candidates for removal.

Properties:

$$A_{low} \subseteq A$$

- If $\delta = 0$, then $A_{low} = \emptyset$ (no parameter is removed for low significance)
- If $\delta = 1$, then $A_{low} = A \backslash \{e : \sigma(e) = 1\}$ (all parameters except those with perfect significance are removed)

Step 3: Identify Redundant Parameter Pairs

The set of redundant parameters $A_{redundant}$ is constructed by examining all pairs of parameters $(e_j) \in A \times A$ with $i < j$.

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Step 3.1: Compute Pairwise Distance

For each pair (e_j), the distance $d(e_j)$is defined as the sum of absolute differences of their graded membership values across all objects:

$$d(e_j) = \sum_{x \in U} |\mu_F(e_i)(x) - \mu_F(e_j)(x)|,\ \forall (e_j) \in A \times A,\ i < j$$

Expanded form:

$$d(e_j) = \sum_{k=1}^{n} |\mu_F(e_i)(x_k) - \mu_F(e_j)(x_k)|$$

Bounds:

Since $0 \leq \mu_F(e)(x) \leq 1$for all $e \in A$and $x \in U$:

$$0 \leq d(e_i, e_j) \leq n,\ \forall (e_i, e_j) \in A \times A$$

where:

- $d(e_j) = 0$if and only if $\mu_F(e_i)(x) = \mu_F(e_j)(x)$for all $x \in U$
- $d(e_j) = n$if and only if $\{(e_i)(x)\mu_F(e_j)(x)\} = \{1\}$for all $x \in U$

Matrix Representation:

The pairwise distance can be computed using the distance matrix. $D = [d_{ij}]_{m \times m}$:

$$d_{ij} = \sum_{k=1}^{n} |m_{ki} - m_{kj}|$$

where $m_{ki} = \mu_F(e_i)(x_k)$.

Step 3.2: Identify Redundant Pairs

A pair $(e_j)$is considered redundant if:

$$d(e_i, e_j) < \varepsilon$$

Step 3.3: Select Parameter to Remove

For each redundant pair $(e_j)$with $d(e_j) <$ ε, the parameter with lower significance is removed:

$$If\ \sigma(e_i) < \sigma(e_j)\ then\ A_{redundant} = A_{redundant} \cup \{e_i\} Else\ A_{redundant} = A_{redundant} \cup \{e_j\}$$

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Formal Definition:

$$A_{\text{redundant}} = \bigcup_{\substack{(e_i, e_j) \in A \times A \\ i < j \\ d(e_i, e_j) < \varepsilon}} \begin{cases} \{e_i\} & \text{if } \sigma(e_i) < \sigma(e_j) \\ \{e_j\} & \text{if } \sigma(e_i) \geq \sigma(e_j) \end{cases}$$

Interpretation:

When two parameters provide nearly identical information (distance below threshold ε), the less significant parameter is removed to eliminate redundancy while preserving the more informative parameter.

Properties:

$$A_{redundant} \subseteq A$$

If $\varepsilon = 0$, then only perfectly identical parameters are considered redundant

If $\varepsilon = n$, then all parameter pairs are considered redundant

Step 4: Compute Reduced Parameter Set

The reduced parameter set A^* is defined as:

$$A^* = A \backslash \left(A_{low} \cup A_{redundant} \right)$$

Expanded form:

$$A^* = \{e \in A : \sigma(e) \geq \delta\} \backslash A_{redundant}$$

Alternative Form:

$$A^* = \left\{ e \in A : \sigma(e) \geq \delta \text{ and } e \notin A_{redundant} \right\}$$

Properties:

$$A^* \subseteq A$$

$$A^* \cap A_{low} = \emptyset$$

$$A^* \cap A_{redundant} = \emptyset$$

If $\delta = 0$ and $\varepsilon = 0$, then $A^* = A$ (no reduction)

If $\delta = 1$ and $\varepsilon = n$, then $A^* = \emptyset$ (complete reduction)

Step 5: Return Reduced Set

Return the reduced parameter set:

$$A^* = \left\{ e \in A : \sigma(e) \geq \delta \text{ and } e \notin A_{redundant} \right\}$$

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Theorem 3.3 (Reduction Preservation). If Algorithm 6.3 removes parameter e, the relative ranking of alternatives under the Graded Weighted Ranking Algorithm remains unchanged, provided that the removed parameter is either low-significance or redundant with a more significant parameter.

Proof: If e is low-significance ($\sigma(e) < \delta$), its contribution to the weighted scores is bounded by $\delta \cdot w_e$. For sufficiently small δ, this contribution cannot alter the relative ordering of alternatives. If e is redundant with e'(distance < ε), then for any alternative u_i, $|\mu_F(e)(u_i) - \mu_F(e')(u_i)| < \varepsilon$. The contribution of $e$to the net score is approximately equal to the contribution of e', so removing $e$preserves the relative ranking.

4. Illustrative Examples

To demonstrate the effectiveness of the proposed approach, we present two detailed illustrative examples: a supplier selection problem and a medical diagnosis scenario. Each example includes complete numerical calculations.

4.1 Supplier Selection Problem

Consider a manufacturing company that must select a supplier from five candidates: $U = \{S_5\}$. The decision is based on four parameters: $E = \{e_4\}$, where:

- e_1: Product quality (benefit criterion)
- e_2: Delivery reliability (benefit criterion)
- e_3: Cost competitiveness (benefit criterion)
- e_4: Technical capability (benefit criterion)

The parameter weights are specified as $W = (0.15)$.

Step 1: Construct Graded Soft Decision Matrix

Decision-makers provide graded evaluations for each supplier-parameter pair. The graded decision matrix $M$is:

Supplier	e_1	e_2	e_3	e_4
S_1	0.85	0.70	0.60	0.90
S_2	0.65	0.90	0.75	0.70
S_3	0.95	0.55	0.45	0.80
S_4	0.70	0.80	0.85	0.60
S_5	0.80	0.65	0.70	0.75

Step 2: Compute Weighted Graded Scores

For each supplier, compute $r_i = \sum_{j=1}^{4} w_j \cdot m_{ij}$:

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$$r_1 = 0.35(0.85) + 0.25(0.70) + 0.25(0.60) + 0.15(0.90) = 0.2975 + 0.1750 + 0.1500 + 0.1350$$

$$r_2 = 0.35(0.65) + 0.25(0.90) + 0.25(0.75) + 0.15(0.70) = 0.2275 + 0.2250 + 0.1875 + 0.1050 = 0.7450$$

$$r_3 = 0.35(0.95) + 0.25(0.55) + 0.25(0.45) + 0.15(0.80) = 0.3325 + 0.1375 + 0.1125 + 0.1200 = 0.7025$$

$$r_4 = 0.35(0.70) + 0.25(0.80) + 0.25(0.85) + 0.15(0.60) = 0.2450 + 0.2000 + 0.2125 + 0.0900 = 0.7475$$

$$r_5 = 0.35(0.80) + 0.25(0.65) + 0.25(0.70) + 0.15(0.75) = 0.2800 + 0.1625 + 0.1750 + 0.1125 = 0.7300$$

Step 3: Compute Weighted Inverse Graded Scores

For each supplier, compute $q_i = \sum_{j=1}^{4} w_j \cdot \left(1 - m_{ij}\right)$:

$$q_1 = 0.35(0.15) + 0.25(0.30) + 0.25(0.40) + 0.15(0.10) = 0.0525 + 0.0750 + 0.1000 + 0.0150$$

$$q_2 = 0.35(0.35) + 0.25(0.10) + 0.25(0.25) + 0.15(0.30) = 0.1225 + 0.0250 + 0.0625 + 0.0450 = 0.2550$$

$$q_3 = 0.35(0.05) + 0.25(0.45) + 0.25(0.55) + 0.15(0.20) = 0.0175 + 0.1125 + 0.1375 + 0.0300 = 0.2975$$

$$q_4 = 0.35(0.30) + 0.25(0.20) + 0.25(0.15) + 0.15(0.40) = 0.1050 + 0.0500 + 0.0375 + 0.0600 = 0.2525$$

$$q_5 = 0.35(0.20) + 0.25(0.35) + 0.25(0.30) + 0.15(0.25) = 0.0700 + 0.0875 + 0.0750 + 0.0375 = 0.2700$$

Step 4: Compute Weighted Net Scores

For each supplier, compute $t_i = r_i - q_i$:

- $t_1 = 0.7575 - 0.2425 = 0.5150$
- $t_2 = 0.7450 - 0.2550 = 0.4900$
- $t_3 = 0.7025 - 0.2975 = 0.4050$
- $t_4 = 0.7475 - 0.2525 = 0.4950$
- $t_5 = 0.7300 - 0.2700 = 0.4600$

Step 5: Rank Alternatives

Ranking in decreasing order of t_i:

1. S_1: $t_1 = 0.5150$
2. S_4: $t_4 = 0.4950$
3. S_2: $t_2 = 0.4900$
4. S_5: $t_5 = 0.4600$
5. S_3: $t_3 = 0.4050$

Step 6: Select Optimal Supplier

The optimal supplier is S_1 with a net score $t_1 = 0.5150$. Supplier S_1 demonstrates the highest overall performance, particularly excelling in product quality (e_1) and technical capability (e_4).

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4.2 Medical Diagnosis Problem

Consider a medical diagnosis scenario where a physician must evaluate four patients: $U = \{P_4\}$for the presence of a particular disease. The diagnostic parameters are $E = \{e_4\}$, representing:

- e_1: Fever severity
- e_2: Fatigue level
- e_3: Cough frequency
- e_4: Headache intensity

The parameter weights reflect diagnostic importance: $W = (0.20)$.

Graded Soft Decision Matrix

Patient	e_1	e_2	e_3	e_4
P_1	0.90	0.80	0.35	0.60
P_2	0.50	0.55	0.85	0.70
P_3	0.75	0.90	0.65	0.45
P_4	0.55	0.65	0.45	0.90

Computation of Net Scores

- $r_1 = 0.35(0.90) + 0.25(0.80) + 0.20(0.35) + 0.20(0.60) = 0.315 + 0.200 + 0.070 + 0.120 =$
- $q_1 = 0.35(0.10) + 0.25(0.20) + 0.20(0.65) + 0.20(0.40) = 0.035 + 0.050 + 0.130 + 0.080 = 0.295$
- $t_1 = 0.705 - 0.295 = 0.410$
- $r_2 = 0.35(0.50) + 0.25(0.55) + 0.20(0.85) + 0.20(0.70) = 0.175 + 0.1375 + 0.170 + 0.140 = 0.6225$
- $q_2 = 0.35(0.50) + 0.25(0.45) + 0.20(0.15) + 0.20(0.30) = 0.175 + 0.1125 + 0.030 + 0.060 = 0.3775$
- $t_2 = 0.6225 - 0.3775 = 0.2450$
- $r_3 = 0.35(0.75) + 0.25(0.90) + 0.20(0.65) + 0.20(0.45) = 0.2625 + 0.225 + 0.130 + 0.090 = 0.7075$
- $q_3 = 0.35(0.25) + 0.25(0.10) + 0.20(0.35) + 0.20(0.55) = 0.0875 + 0.025 + 0.070 + 0.110 = 0.2925$
- $t_3 = 0.7075 - 0.2925 = 0.4150$
- $r_4 = 0.35(0.55) + 0.25(0.65) + 0.20(0.45) + 0.20(0.90) = 0.1925 + 0.1625 + 0.090 + 0.180 = 0.6250$
- $q_4 = 0.35(0.45) + 0.25(0.35) + 0.20(0.55) + 0.20(0.10) = 0.1575 + 0.0875 + 0.110 + 0.020 = 0.3750$
- $t_4 = 0.6250 - 0.3750 = 0.2500$

Ranking

1. $P_3: t_3 = 0.4150$
2. $P_1: t_1 = 0.4100$
3. $P_4: t_4 = 0.2500$
4. $P_2: t_2 = 0.2450$

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Patients $P_3$and P_1show the highest symptom scores, with P_3marginally higher. The physician should prioritise diagnostic testing for these patients.

4.3 Sensitivity Analysis

To assess the robustness of the decision, we perform sensitivity analysis by varying the parameter weights.

Scenario A: Equal Weights ($W = (0.25)$)

For the supplier selection problem:

- $t_1 = 0.25(3.05) - 0.25(0.95) = 0.7625 - 0.2375 = 0.5250$
- $t_2 = 0.25(3.00) - 0.25(1.00) = 0.7500 - 0.2500 = 0.5000$
- $t_3 = 0.25(2.75) - 0.25(1.25) = 0.6875 - 0.3125 = 0.3750$
- $t_4 = 0.25(2.95) - 0.25(1.05) = 0.7375 - 0.2625 = 0.4750$
- $t_5 = 0.25(2.90) - 0.25(1.10) = 0.7250 - 0.2750 = 0.4500$

Ranking: $S_1 > S_2 > S_4 > S_5 > S_3$(changed)

Scenario B: Quality-Focused Weights ($W = (0.15)$)

- $t_1 = 0.50(0.85) + 0.20(0.70) + 0.15(0.60) + 0.15(0.90) - [0.50(0.15) + 0.20(0.30) + 0.15(0$
- $t_1 = (0.425 + 0.140 + 0.090 + 0.135) - (0.075 + 0.060 + 0.060 + 0.015) = 0.790 - 0.210 = 0.580$
- $t_2 = (0.325 + 0.180 + 0.1125 + 0.105) - (0.175 + 0.020 + 0.0375 + 0.045) = 0.7225 - 0.2775 = 0.445$
- $t_3 = (0.475 + 0.110 + 0.0675 + 0.120) - (0.025 + 0.090 + 0.0825 + 0.030) = 0.7725 - 0.2275 = 0.545$
- $t_4 = (0.350 + 0.160 + 0.1275 + 0.090) - (0.150 + 0.040 + 0.0225 + 0.060) = 0.7275 - 0.2725 = 0.455$
- $t_5 = (0.400 + 0.130 + 0.105 + 0.1125) - (0.100 + 0.070 + 0.045 + 0.0375) = 0.7475 - 0.2525 = 0.495$

Ranking: $S_1 > S_3 > S_5 > S_4 > S_2$(changed)

The sensitivity analysis demonstrates that the ranking is sensitive to weight variations, which is expected given the different strengths of the suppliers. This analysis provides valuable information for decision-makers regarding the robustness of their choices.

Comparative Analysis

To evaluate the effectiveness of the proposed approach, we compare it with existing methods for decision-making under uncertainty.

5.1 Comparison with Classical Soft Sets

Classical soft set theory, with its binary membership representation, cannot capture the graded nature of uncertainty that characterises the examples presented in Section 5.5. In the supplier selection problem, for instance, classical soft sets would require threshold-based dichotomisation of membership degrees, potentially lose valuable information and produce less nuanced rankings.

Aspect	Classical Soft Sets	Graded Soft Sets
Membership	Binary {0, 1}	Graded [0, 1]
Information Loss	High	Low
Uncertainty Handling	Limited	Comprehensive

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Computational Complexity	Low	Moderate

5.2 Comparison with Fuzzy Soft Sets

Fuzzy soft sets extend classical soft sets by incorporating membership degrees in [0, 1], providing similar expressive capability to the proposed framework. However, fuzzy soft sets implicitly capture non-membership through complement operations, whereas Graded Soft Set Theory provides an explicit dual perspective through inverse graded membership functions.

Aspect	**Fuzzy Soft Sets**	**Graded Soft Sets**
Membership Representation	Point value in [0, 1]	Point value in [0, 1]
Inverse Membership	Implicit	Explicit
Decision Framework	Single perspective	Dual perspective
Theoretical Complexity	Moderate	Moderate-Low

5.3 Comparison with Intuitionistic Fuzzy Soft Sets

Intuitionistic fuzzy soft sets characterise each element-parameter relationship through both membership and non-membership degrees, providing a dual perspective similar to the proposed framework. However, intuitionistic fuzzy soft sets require the specification of both membership and non-membership functions independently, which increases cognitive burden and computational complexity.

Aspect	**Intuitionistic Fuzzy Soft Sets**	**Graded Soft Sets**
Membership Functions	Two independent functions	One function + complement
Cognitive Burden	High	Moderate
Computational Complexity	High	Moderate
Constraint	$\mu + \nu \leq 1$	$\mu + \nu = 1$

5.4 Comparison with Recent Hybrid Models

Recent hybrid models, such as Intuitionistic Fuzzy Rough Soft (IFRS) sets, integrate the advantages of intuitionistic fuzzy sets, rough sets, and soft sets to effectively handle uncertainty, vagueness, and ambiguity. While these models offer enhanced expressive power, they introduce significant theoretical complexity and computational overhead.

Aspect	**IFRS Sets**	**Graded Soft Sets**
Theoretical Components	Three theories	One theory extension
Computational Complexity	High	Moderate

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Implementation Difficulty	High	Moderate
Expressive Power	Very High	High

The proposed Graded Soft Set Theory offers a balanced approach that enhances the expressive power of classical soft sets without requiring the full machinery of more complex hybrid models.

6. Conclusion

In this paper, a novel solution of soft set theory in decision-making under uncertainty is presented. In classical soft sets, membership is represented in binary form, which is a fundamental limitation of classical soft sets. The proposed Graded Soft Set Theory works on the idea of using graded membership functions and inverse graded membership functions to enable graded membership degrees. This extension will maintain the parameterised framework of the soft sets and will greatly expand their expressive power.

A set of formal definitions and operations for graded soft sets and fundamental properties are provided in the theoretical aspects of this chapter, while three decision-making algorithms are developed. The illustrative examples provide concrete examples of how the framework can be used to solve real-world decision problems, and the comparative analysis places the contribution in the context of the other uncertainty theories.

The proposed approach is a significant step forward in the quest for establishing mathematical models that can reflect the complex uncertainty involved in practical decision-making. Graded Soft Set Theory thus provides a compromise of the two extremes, namely, the binary simplicity of classical soft sets and the complexity of fully graded uncertainty representation, building a balanced and practical tool for decision-makers in an uncertain world.

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