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Analysis of forced vibration of simply supported functionally graded nonlocal Euler–Bernoulli beams

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Abstract

In this study, the forced vibration of a simply supported Euler-Bernoulli beam is investigated. Small-scale effects with a nonlocal parameter are taken into consideration in a nonlocal beam model. The vibration amplitude corresponding to local and nonlocal effects are analyzed under the simply supported boundary conditions. An amplitude ratio between the non-local and local responses is derived to observe the impact of the non-local parameter and excitation frequency on the dynamic behavior of the beam. The effect of modal number and nonlocal parameter on the forced vibration response is examined. The variation of amplitude ratio with frequency ratio for different values of modal number and non-local parameter are shown graphically.

Keywords: Forced Vibration, Amplitude Ratio, Non-Local Parameter, Excitation Frequency, Euler-Bernoulli Beam

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1. INTRODUCTION

Beams structural components and vibrational behavior are an important factors when they come about the design and operations subjected to various loads as an engineering application. Euler-Bernoulli beam theory is one of the most popular classical theories for the analysis of transverse vibration. In Euler-Bernoulli beam models use assumptions from local continuum mechanics to describe the mechanical behavior of the beam. This approach is effective for conventional macro-scale structures, but its use is limited when a structure's distinctive dimensions become small enough. At micro and nano dimensions, size-dependent effects become significant. The traditional local elasticity theory is not sufficient to describe the mechanical response. Such small-scale effects have been explained by nonlocal elasticity theory, in which the stress at a point depends not only on the strain at that site but also on the deformation of the surrounding material. Consequently, nonlocal theories offer a framework for incorporating size-dependent behavior into continuum models.

Free vibration, natural frequencies, and size-dependent properties of nonlocal and functionally graded beams have received a lot of attention but forced vibration is equally important because the amplitude of vibrations directly depend on the relationship between excitation frequency and the structure's natural frequency. In particular, comparing the local and nonlocal forced vibration amplitudes is a useful method to assess the effect of nonlocality on the dynamic response. The amplitude ratio is a dimensionless representation of this influence that facilitates direct comparison of the two beam models. The several researchers have used various analytical techniques to examine the vibration characteristics of beam structures. These investigations include how different boundary conditions, shear deformation, rotary inertia, and material inhomogeneity affect beams' natural frequencies, mode forms, and dynamic responses [Abu-Hilal (2003), Reddy (2007), Reddy and Pang (2008), and Aydogdu (2009)]. Karaoglu and Aydogdu (2010) used both local and non-local Euler-Bernoulli beam theory to investigate the forced vibration of carbon nanotubes. They analyzed the amplitude ratio for the local and non-local Euler-Bernoulli beam models for single and double-walled carbon nanotubes. Simsek (2010) evaluated the forced vibration response of a simply supported single-walled carbon nanotube under a moving harmonic load.

In order to study the bending, buckling, and vibration of nanobeams, Thai (2012) created a nonlocal shear deformation beam theory that incorporates small-scale effects and quadratic fluctuation of shear stresses without requiring a shear correction factor. Repetto et al. (2012) investigated the theoretical and experimental vibration properties of a cantilever beam with its clamped end above the free end under transverse stimulation at a specified frequency. Uymaz (2013) examined the forced vibration behaviour of functionally graded nanobeams and find the solutions using the Navier method. Sioma (2015) suggested a measurement technique that used contactless optical sensors to analyze the forced vibrations of cantilever beams. Yang et al. (2016) incorporated a meshfree boundary-domain integral equation approach to examine the forced vibration behavior of two-dimensional functionally graded beams. Chen et al. (2016) explored the free and forced vibration properties of functionally graded porous beams with non-uniform porosity distributions using transverse shear deformation and Timoshenko beam theory. Kim et al. (2017) provided a thorough modal analysis of simply supported Timoshenko beams, including natural frequencies and three types of mode shapes. They predicted structural dynamic reactions using the modal analysis approach. Akbas (2018) assessed the forced vibration behavior of functionally graded porous deep beams under dynamic loading by taking into account the variation of mechanical characteristics and porosity along the thickness.

The contribution on the several beam theories and constitutive models to investigate forced vibration, frequency-response behavior, and the impacts of non-locality and material gradation are documented by El-Borgi et al. (2015), Simsek (2019), and Trabelssi et al. (2019). Mahapatra and Panigrahi (2019) used

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linear and torsional springs to analyze the free vibrations of beams with elastically constrained end conditions. Jrad et al. (2020) analyzed the free and forced vibration behavior along with natural frequencies, mode shapes, and steady-state dynamic responses under various boundary conditions using analytical solutions and a seven-degree-of-freedom finite element model. Elmeiche et al. (2020) explored the forced vibration behavior of functionally graded beams subjected to moving harmonic loads. Simsek and Kocaturk (2020) studied the forced vibration and free vibration behaviour of a functionally graded beam under concentrated moving harmonic load with simply supported boundary conditions. Kilic and Ozdemir (2021) analyzed the dynamic and stability behaviour of axially functionally graded beams with different material properties over their length. They created the finite element models using first-order shear deformation theory and classical beam theory. Civalek et al. (2021) presented the forced vibration analysis of a simply supported composite beam exposed to a harmonic point load at the beam's midpoint using the first-order shear deformation beam theory. Nazmul et al. (2023) employed Hamilton's principle, Euler-Bernoulli and Timoshenko beam theories to provide analytical vibration solutions for bidirectional functionally graded nanobeams. Yavuz et al. (2025) examined the dynamic properties of uniform-cross-section functionally graded beams under different boundary conditions, slenderness ratios, temperature-dependent material properties, and thermal environments. They used the generalized differential quadrature technique to solve the governing equations, which are constructed using Hamilton's principle within the Euler-Bernoulli beam theory.

The primary focus of this study is to find the analytical solution of the forced vibration of a simply supported Euler-Bernoulli beam while accounting for nonlocal effects. The governing equation is developed using a nonlocal Euler-Bernoulli beam model, and harmonic excitation is considered. Analytical formulas for the vibration amplitudes corresponding to the local and nonlocal formulations are derived. The amplitude ratio between the nonlocal and local responses is determined using the excitation frequency and the corresponding local natural frequency. The effect of the modal number and nonlocal parameter on the forced vibration response is shown graphically.

2. MATHEMATICAL FORMULATION

Consider a simply supported, functionally graded Euler-Bernoulli beam with dimensions L, b and h into the Cartesian coordinates loaded in transverse direction by a distributed load q.

The transverse displacement of any point of the beam w, and the axial displacement, u, are determined by the Euler-Bernoulli beam theory.

$$\begin{cases} u = -z\dfrac{\partial w}{\partial x} \\ w = w(x,t) \end{cases}, \tag{1}$$

where the mid-plane axial displacement is denoted by u_0. Assuming minor deformations, the Euler-Bernoulli beam theory's sole nonzero strain is

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = -z\frac{\partial^2 \delta w}{\partial x^2}. \tag{2}$$

Eringen's non-local constitutive equation can be given as

$$\left(1-\mu\nabla^2\right)\sigma^{nl} = C:\varepsilon,\ \mu = (e_0 a)^2. \tag{3}$$

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The relationship between the local and non-local stresses for a non-local beam can be described as

$$\sigma_{xx} - \mu \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E\varepsilon_{xx}. \tag{4}$$

The idea of Hamilton's principle can be used to generate the governing equations of the FGM Euler-Bernoulli beam subjected to dynamic load. That's

$$\int_{t_1}^{t_2} (\delta U - \delta k) dt = 0, \tag{5}$$

where δU and δk stand for the strain energy and kinetic energy variation, respectively. Consequently, the strain energy fluctuation can be computed as

$$\begin{aligned} \int_{t_1}^{t_2} \delta U dt = \int_{t_1}^{t_2} \int_v \sigma_{xx} \delta\varepsilon_{xx} dV dt = \int_{t_1}^{t_2} \int_v \sigma_{xx} \left(\frac{\partial \delta u_0}{\partial x} - z \frac{\partial^2 \delta w}{\partial x^2} \right) dV dt \\ = \int_{t_1}^{t_2} \int_0^L \left[-\frac{\partial N_{xx}}{\partial x} \delta u_o - \frac{\partial^2 M_{xx}}{\partial x^2} \delta w \right] dx dt, \end{aligned} \tag{6}$$

where

$$N_{xx} = \int_A \sigma_{xx} dA, \tag{7}$$

and

$$M_{xx} = \int_A z\sigma_{xx} dA. \tag{8}$$

The kinetic energy variation can be expressed in the following way:

$$\delta \int_{t_1}^{t_2} k dt = \delta \left\{ \frac{1}{2} \int_{t_1}^{t_2} \int_V \rho \left[\left(\frac{\partial u}{\partial t} \right)^2 + \left(\frac{\partial w}{\partial t} \right)^2 \right] dV dt \right\} = \int_t^{t_2} \int_0^L \left[m_2 \frac{\partial^4 w}{\partial x^2 \partial t^2} - m_0 \frac{\partial^2 w}{\partial t^2} \right] \delta w dx dt. \tag{9}$$

On substituting equations (6) and (9) into equation (5), we get the governing equations as follows:

$$\frac{\partial N_{xx}}{\partial x} = 0 \tag{12}$$

and

$$\frac{\partial^2 M_{xx}}{\partial x^2} = m_0 \frac{\partial^2 w}{\partial t^2} - m_2 \frac{\partial^4 w}{\partial x^2 \partial t^2} + q, \tag{13}$$

where q is the transverse distributed load along x-axis.

Multiplying equation (13) by z and the resulting equation is then integrated over the area of the beam cross section A, we get

$$M_{xx} - \mu \frac{\partial^2 M_{xx}}{\partial x^2} = \int_A Ez \left(-z \frac{\partial^2 w}{\partial x^2} \right) dA = -I_2 \frac{\partial^2 w}{\partial x^2}, \tag{14}$$

it can also be expressed as

$$M_{xx} - \mu \frac{\partial^2 M_{xx}}{\partial x^2} = -I_2 \frac{\partial^2 w}{\partial x^2}. \tag{15}$$

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On taking the second derivative of equation (15), we obtain

$$\frac{\partial^2 M_{xx}}{\partial x^2} - \mu \frac{\partial^4 M_{xx}}{\partial x^4} = -\left[I_2 \frac{\partial^4 w}{\partial x^4} \right]. \tag{16}$$

Now substituting equation (13) into equation (16), the Navier's equation can be expressed as

$$m_0 \frac{\partial^2 w}{\partial t^2} + \left\{ -(m_2 + \mu m_0) \right\} \frac{\partial^4 w}{\partial x^2 \partial t^2} + \mu m_2 f_2(x) \frac{\partial^6 w}{\partial x^4 \partial t^2} + I_2 \frac{\partial^4 w}{\partial x^4} = \mu \frac{\partial^2 q}{\partial x^2} - q \,, \tag{17}$$

where

$$\mu = (e_0 a)^2, m_0 = b\int_{-h/2}^{h/2} g_2(z)\, dz,\ m_2 = b\int_{-h/2}^{h/2} z^2 g_2(z)\, dz. \tag{18}$$

The displacement is represented using the same spatial and temporal harmonic functions as the excitation in order to determine the forced-vibration response

$$w(x,t) = \sum_{m=1}^{\infty} W_m \sin\left(\frac{m\pi x}{L} \right) \sin \omega t \,, \tag{19}$$

and harmonic load applied to the beam can be defined as

$$q(x,t) = \sum_{m=1}^{\infty} q_m \sin\left(\frac{m\pi x}{L} \right) \sin \omega t. \tag{20}$$

2.1 Boundary conditions

The simply supported boundary conditions can be written as

$$w = M = 0, \quad \text{at } x = 0 \text{ and } x = L \,. \tag{21}$$

2.2 Non-local modal amplitude

The modal representation is substituted into the chosen nonlocal governing equation, the terms corresponding to the m mode are collected and we get

$$\left[\omega^2 \left\{ m_0 + \left(m_2 + \mu m_0 \right) K_m^2 + \mu m_2 K_m^4 \right\} - K_m^4 I_2 \right] W = \left(1 + \mu K_m^2 \right) q_0 \,, \tag{22}$$

and the non-local amplitude becomes

$$W_m^{nl} = \frac{\left(1 + \mu K_m^2 \right) q_0}{\omega^2 \left(m_0 + m_2 K_m^2 \right) \left(1 + \mu K_m^2 \right) - I_2 K_m^4} \,. \tag{23}$$

2.4 Local Modal amplitude

The local beam model is obtained by setting the nonlocal parameter $\mu = 0$ in equation (22), the modal equation (22) becomes

$$\left[\omega^2 \left\{ m_0 + m_2 K_m^2 \right\} - I_2 K_m^4 \right] W = q_0 \,, \tag{24}$$

and the local amplitude can be written as

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$$W_m^l = \frac{q_0}{\omega^2 \left\{ m_0 + m_2 K_m^2 \right\} - I_2 K_m^4} \quad (25)$$

2.5 Amplitude ratio

To compare the responses predicted by the two formulations, the ratio between the nonlocal and the local amplitudes is defined as

$$f(r) = \frac{W_m^{nl}}{W_m^l} \quad (26)$$

Substituting the values of equations (23) and (25) in equation (26), we get

$$f(r) = \frac{(1+\mu K_m^2)\left\{\omega^2 \left(m_0 + m_2 K_m^2\right) - I_2 K_m^4\right\}}{\omega^2 (m_0 + m_2 K_m^2)(1+\mu K_m^2) - I_2 K_m^4} \quad (27)$$

Equation (27) maintains the dimensional excitation frequency while comparing the local and nonlocal responses.

The local natural frequency from the local modal equation is defined as

$$(\omega_m^l)^2 = \frac{I_2 K_m^4}{m_0 + m_2 K_m^2} \quad (28)$$

The dimensionless excitation-to-natural-frequency ratio is then introduced as

$$r = \frac{\omega}{\omega_m^l} \quad (29)$$

Using equations (28) and (29) into equation (27), we get the amplitude ratio for local and non-local Euler Bernoulli beam theories as

$$f(r) = \frac{(1-r^2)\left(1+\mu K_m^2\right)}{1-r^2\left(1+\mu K_m^2\right)}, \quad (30)$$

it can also be written as

$$f(r) = \frac{(1-r^2)\kappa}{1-r^2\kappa}, \quad (31)$$

where $K_m = \left(\frac{m\pi}{L}\right)$ is modal wave number and $\kappa = \left(1+\mu K_m^2\right)$ is the nonlocal contribution.

3. RESULTS AND DISCUSSION

The results are shown how the modal number and nonlocal parameter affect the beam's forced vibration response. The calculated nonlocal-to-local amplitude ratio serves as the foundation for the analysis. The beam length is kept constant throughout the analysis, but the frequency ratio, nonlocal parameter, and modal number are varied. The analytical expression is obtained in the previous section is used to calculate the amplitude ratio. The curves are shown the nonlocal effects and the dependence of the relative vibration

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amplitude on the stimulation frequency.

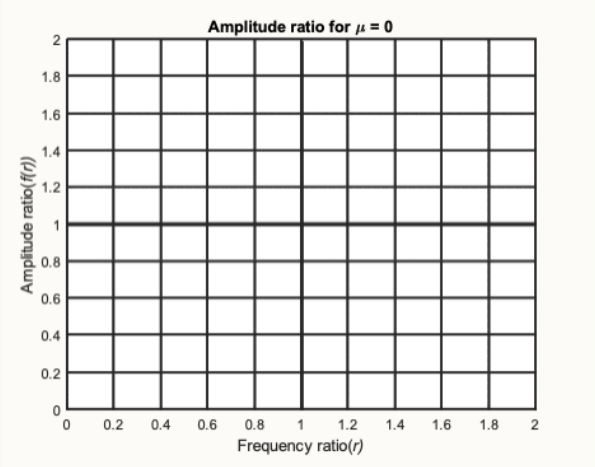


Fig. 1(a): Variation of amplitude ratio with frequency ratio for $\mu = 0$

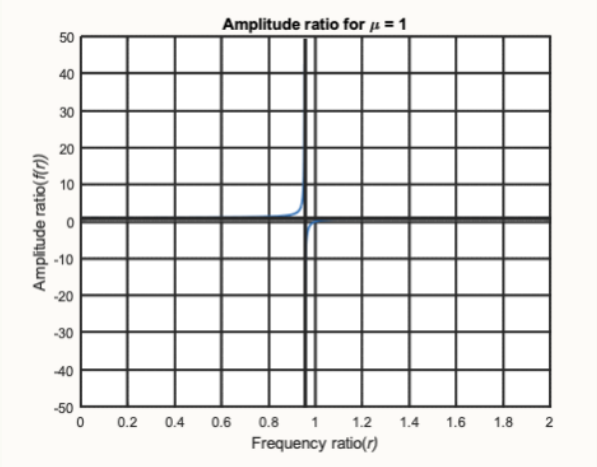


Fig. 1(b): Variation of amplitude ratio with frequency ratio for $\mu = 1$

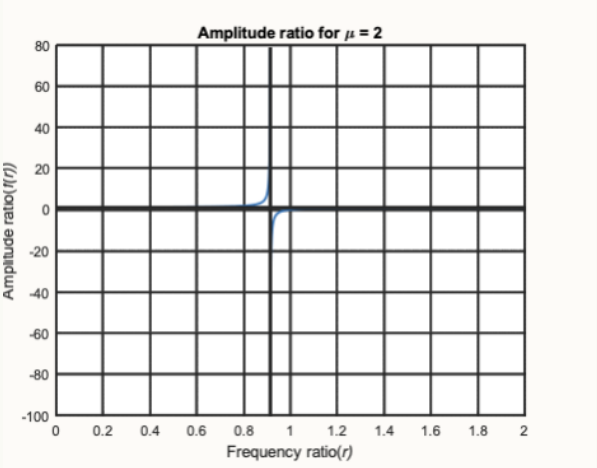


Fig. 1(c): Variation of amplitude ratio with frequency ratio for $\mu = 2$

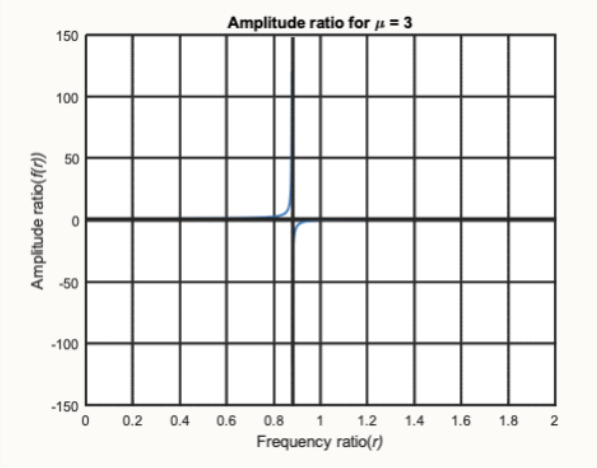


Fig. 1(d): Variation of amplitude ratio with frequency ratio for $\mu = 3$

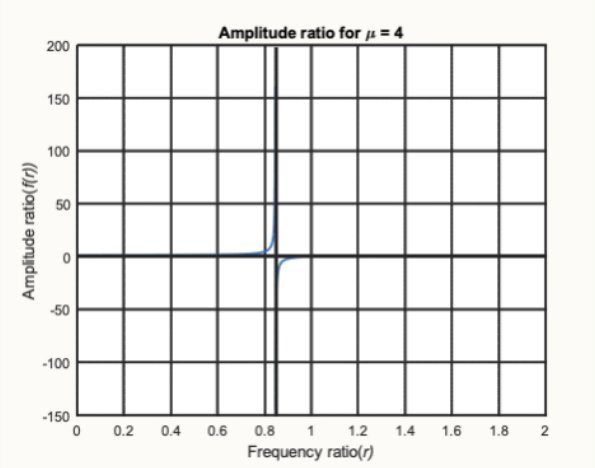


Fig. 1(e): Variation of amplitude ratio with frequency ratio for $\mu = 4$

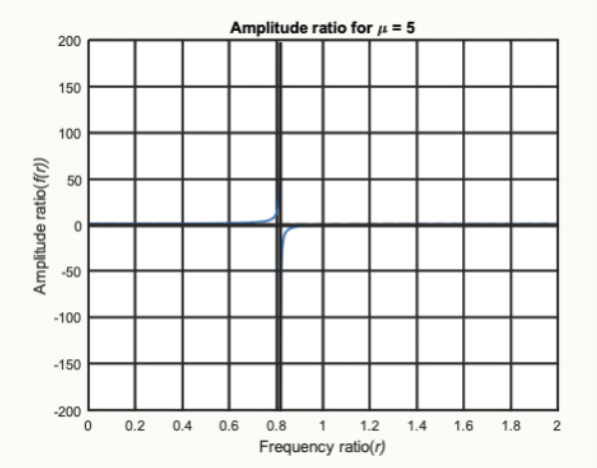


Fig. 1(f): Variation of amplitude ratio with frequency ratio for $\mu = 5$

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Figures 1(a)-(f) show how the nonlocal-to-local amplitude ratio varies with the frequency ratio for different values of nonlocal parameter. The beam length is fixed at $L = 10$ and the initial vibration mode $m = 1$ is considered throughout the analysis. The local and nonlocal formulations coincide in the absence of nonlocal effects $(\mu = 0)$ as shown in Fig. 1(a). The amplitude ratio remains equal to unity throughout the frequency range under consideration [similar nonlocal effects on the relative forced-vibration response were found by Karaoglu and Aydogdu (2010)] whereas the amplitude ratio changes with the frequency ratio and shows a singularity at a specific frequency ratio for nonzero values of the nonlocal parameter. This singularity is equivalent to the denominator of the resulting amplitude-ratio expressions. The singularity moves toward lower frequency ratio as the nonlocal parameter rises. This behavior is consistent with the mathematical expression of the singular frequency ratio, which depends on both the nonlocal parameter and the modal wave number. The curves also show changes in the sign and magnitude of the amplitude ratio around the unique point. The amplitude ratio approaches unity away from the singularity as the frequency ratio increases. These results demonstrate that the nonlocal parameter has a significant impact on the frequency-dependent relationship between the local and nonlocal forced-vibration amplitudes.

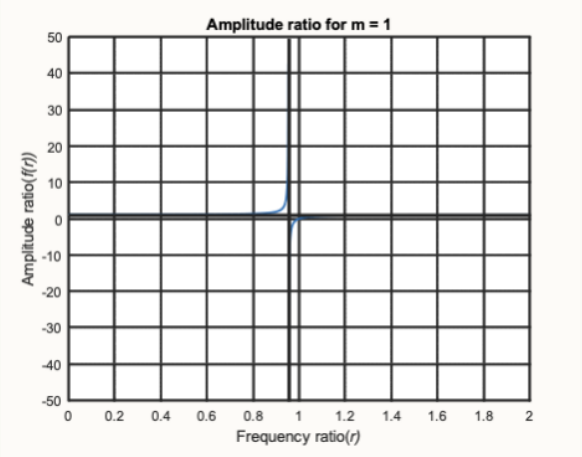


Fig. 2(a): Variation of amplitude ratio with frequency ratio for $m = 1$

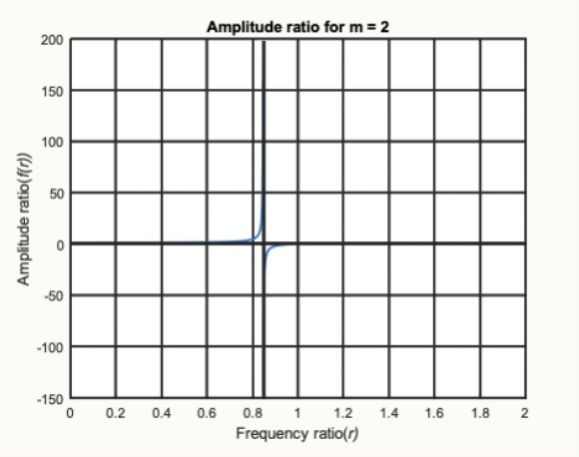


Fig. 2(b): Variation of amplitude ratio with frequency ratio for $m = 2$

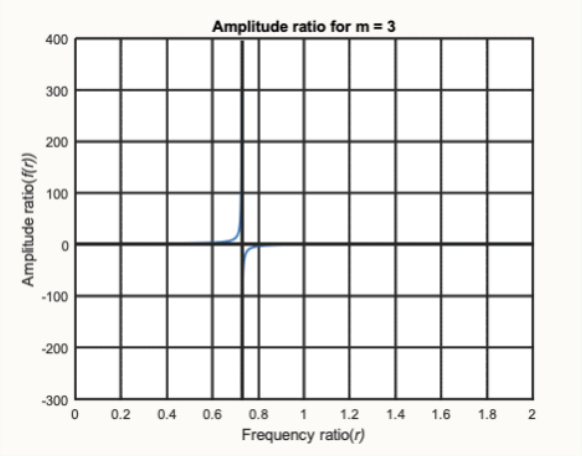


Fig. 2(c): Variation of amplitude ratio with frequency ratio for $m = 3$

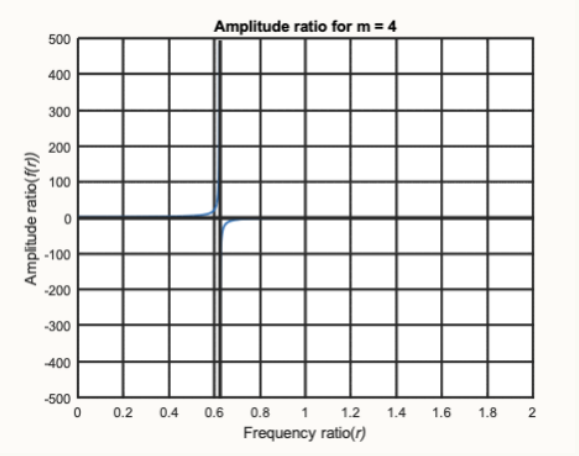


Fig. 2(d): Variation of amplitude ratio with frequency ratio for $m = 4$

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For the values of mode number $m = 1, 2, 3$ and 4, the nonlocal-to-local amplitude ratio varies with the frequency ratio as shown in Figures 2(a)-(d). The beam length and the nonlocal parameter are kept fixed at $L = 10$ and $\mu = 1$. The findings show that the modal number has a significant impact on the amplitude-ratio curves. A shift in the frequency-dependent relationship between the local and nonlocal responses indicate that the singularity shifting toward lower frequency ratios as the modal number grows.

The modal wave number increases with the mode number, thereby amplifying the nonlocal effects at higher modes. As the mode number rises, the nonlocal term's contribution consequently becomes more noticeable. As the modal number rises, the singular frequency ratio consequently falls. The graphs show positive and negative values surrounding the singularity, which corresponds to variations in the amplitude ratio's sign. The amplitude ratio gets close to unity with high frequency ratios. These findings suggest that the impact of nonlocality on the relative forced-vibration response depends more and more on the vibration modes.

4. CONCLUSION

The forced vibration behaviour of simply supported functionally graded Euler-Bernoulli beams taking nonlocal effects into account is examined. Under harmonic excitation, closed-form equations for the local and nonlocal modal amplitudes are obtained. A dimensionless amplitude ratio is introduced to observe the effects of the modal number and nonlocal parameter on the dynamic response. The numerical analysis is carried out for a fixed value of beam length taking into account for various vibration mode and nonlocal parameter values. The findings of this study can be summarized as follows:

- A non-dimensional equation is developed to compare the local and nonlocal vibration responses as a function of the excitation-frequency ratio.
- As the nonlocal parameter grows, the amplitude ratio varies and the singularity moves toward lower frequency ratios.
- As the modal number increases, the singularity moves toward lower frequency ratios showing the mode-dependent influence of nonlocality.
- The amplitude ratio equals to unity wherever it is defined, and the local and nonlocal formulations coincide when the nonlocal parameter is zero.
- The amplitude ratio depends on the modal number for a simply supported functionally graded beam.
- For comparing local and nonlocal forced-vibration responses, the formulation offers a practical analytical framework.

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Conflict of interest

The authors declare no conflict of interest.

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Nomenclature

L	Length of nanobeam	I_0, I_1, I_2	Material property integrals
b	Width of nanobeam	m_0, m_2	Mass inertial coefficients
h	Thickness of nanobeam	t	Time
w	Transverse displacement	u	Axial displacement
E_0	Reference Young's modulus	Ω	Non-dimensional frequency parameter
$g_1(z)$	Material grading function	ε_{xx}	Strain tensor
$W(x)$	Non dimensional transverse displacement function	$X = x/L$	Non-dimensional axial coordinate
x	Axial coordinate along the beam	q	Harmonic load
z	Thickness coordinate	β	Elastic foundation parameter
μ	Nonlocal parameter	ω_m^{nl}	Non-local frequency
$\rho(z)$	Density function	ω_m^{l}	Local frequency
ω	Natural frequency	$F(r)$	Amplitude ratio
$E(z)$	Young's modulus varying along thickness direction	r	Frequency ratio
K_m	Modal wave number	κ	Nonlocal contribution

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