Published by

British Journal of Contemporary Research

BEXFORD PUBLISHING

A Simple Quantitative Measure of Democracy in the Decision-Making Process in a Group

Original Research Article | Volume 1 | Issue 2 | 2026 | Article Number: 038

Accepted: 04 July 2026 | Published: 10 July 2026 | ISSN: 2979-8582 (Online)

Crossref doi : **https://doi.org/10.67693/BJCR-GDG44384**

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Abstract

Quantifying the degree of democracy in the decision-making process of a group is an important problem in political science, organizational studies and social decision making. Existing democracy indices are primarily intended for evaluating countries or institutions and are not designed to measure the level of democratic participation within an individual group. In this work, a simple mathematical model is proposed to quantify the degree of democracy based on the participation indices of group members. An empirical expression is introduced for the distribution of participation levels, from which analytical expressions are derived for the democracy index and the percentage of fully participating members. The proposed model requires only one empirical parameter and is computationally simple to implement. The calculated results show that both the democracy index and the percentage of fully participating members increase monotonically with the model parameter, consistent with the intuitive expectation that greater participation corresponds to a higher degree of democracy. Although the present formulation is preliminary and the adopted participation distribution has certain limitations, the proposed framework provides a simple quantitative basis for analysing and comparing the democratic nature of decision-making processes in different groups and organizations.

Keywords: Democracy Index, Group Decision Making, Participation Index, Mathematical Model, Collective Decision Making, Social Choice

1. Introduction

Decision making in groups is an important aspect of governments, organizations, institutions and social

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communities. In democratic systems, decisions are expected to reflect the participation and opinions of members rather than the preferences of a single individual or a small subset of the group. Consequently, the degree of participation of members has often been regarded as one of the fundamental characteristics of democratic decision making [1, 2].

Several indices have been proposed to evaluate the quality of democracy at national and institutional levels. Examples include the Democracy Index published by the Economist Intelligence Unit [3], the Polity Project [4], and the Varieties of Democracy (V-Dem) Project [5]. These indices employ multiple political, institutional and socio-economic indicators to compare democratic performance across countries. Although such indices are valuable for large-scale political analysis, they are not intended to quantify the degree of democracy in the decision-making process of an individual group or organization.

From a mathematical perspective, the analysis of collective decision making has attracted considerable attention through voting theory, social choice theory and measures of voting power [6-8]. These approaches mainly focus on voting procedures, aggregation of preferences and the distribution of voting power among participants. However, they do not directly provide a simple quantitative measure of the extent to which members actually participate in the decision-making process.

The present work proposes a simple mathematical framework for quantifying the degree of democracy in group decision making based on the participation indices of individual members. The model introduces a democracy index together with the percentage of fully participating members, both of which are obtained from an empirical participation distribution. The formulation is computationally simple, requires only one empirical parameter, and provides a quantitative measure that may be useful for comparing the democratic nature of decision-making processes across different groups and organizations. Although the proposed model is preliminary, it offers a basis for further mathematical refinement and empirical validation.

2. Mathematical Formulation

Consider a group of people comprising $m + 1$ members where one of them is the coordinator whose job is to reach a decision, on any matter, based on the opinions of m members. Ask each member a question: How much say do you have in the decision-making process of your group? In approximately what fraction of decisions is your opinion taken into account? The member should answer these questions based on his or her overall experience and perception of the group's decision-making process over an extended period rather than on a formal record of past decisions.

The response of the ith member should be recorded as a number, say f_i, falling in the range $0 \leq f_i \leq 1$. Based on the results of this survey among members, I propose an index of democracy (D) to be defined by the following equation.

$$D = \frac{1}{m} \sum_{i=1}^{m} f_i \tag{1}$$

Let us call f_i the participation index.

Equation (1) suggests that, $0 \leq D \leq 1$.

For computational convenience, let us assume that f_i has a discrete distribution of values from 0 to 1, in steps of 0.1.

I define $k(y)$ as the number of members having participation index exactly equal to y. Thus, the democracy index (D) is given by,

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$$D = \frac{1}{m}\sum_{y=0}^{1} k(y)y \tag{2}$$

The above summation is over the discrete values $y = 0, 0.1, ..., 1$.

I have chosen an expression for $k(y)$ based on the assumptions that, 1) $k(0) = 0$, 2) $k(y)$ reaches a peak value as a function of y and it decreases beyond that value. It is given by,

$$k(y) = Cy\, exp(-by) \tag{3}$$

where $C, b > 0$.

The value of C can be obtained from the normalization condition that, $\sum_{y} k(y) = m$. Thus one obtains,

$$C = \frac{m}{\sum_{y=0}^{1} y\, exp(-by)} \tag{4}$$

Substituting equation (4) into equation (3), one gets,

$$k(y) = \frac{my\, exp(-by)}{\sum_{y=0}^{1} y\, exp(-by)} \tag{5}$$

The value of y at which $k(y)$ attains its maximum value is denoted here by y_{max}. It can be obtained by setting $\frac{dk(y)}{dy} = C(1 - by)exp(-by) = 0$. Thus, one gets,

$$y_{max} = \frac{1}{b} \tag{6}$$

for the corresponding continuous function $k(y)$.

Although y is sampled at discrete intervals (i.e., $y = 0, 0.1, ..., 1$) in the present model, equation (6) is obtained from the corresponding continuous approximation, which provides the location of the peak of the empirical distribution.

Substituting equation (6) into equation (5), one obtains,

$$k(y) = \frac{my\, exp\left(-y/y_{max}\right)}{\sum_{y=0}^{1} y\, exp\left(-y/y_{max}\right)} \tag{7}$$

According to equation (7), the expected fraction of members whose opinions are involved in approximately $100y$ % of decisions is $\frac{y\, exp\left(-y/y_{max}\right)}{\sum_{y=0}^{1} y\, exp\left(-y/y_{max}\right)}$.

Substituting equation (7) into equation (2), one obtains,

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$$D = \frac{\sum_{y=0}^{1} y^2 \exp\left(-y/y_{max}\right)}{\sum_{y=0}^{1} y \exp\left(-y/y_{max}\right)} \tag{8}$$

Eq. (8) is actually a weighted mean of the participation levels. Therefore, $0 \leq D \leq 1$.

If a survey shows that x% of the members have full participation in decision making (i.e., $f_i = 1$ for them), then from equation (7) one obtains,

$$x = \frac{100 \exp\left(-1/y_{max}\right)}{\sum_{y=0}^{1} y \exp\left(-y/y_{max}\right)} \tag{9}$$

Substituting equation (9) into equation (8), one gets,

$$D = x \frac{\sum_{y=0}^{1} y^2 \exp\left(-y/y_{max}\right)}{100 \exp\left(-1/y_{max}\right)} \tag{10}$$

For a given value of y_{max}, equations (8) and (9) provide the corresponding values of D and x, respectively. A relation between D, x and y_{max} is given by equation (10). I define a new function $u(z)$ as the number of members with $f_i \geq z$. Using equation (7), it is obtained as,

$$u(z) = \sum_{y \geq z}^{1} k(y) = \sum_{y \geq z}^{1} \frac{my \exp\left(-y/y_{max}\right)}{\sum_{y=0}^{1} y \exp\left(-y/y_{max}\right)} \tag{11}$$

The summation in the denominator within the above summation is over the discrete set $y = 0, 0.1, ..., 1$. The outer summation is over the discrete set $y = z, z + 0.1, ..., 1$.

For the analytical model, $u(z)$ represents the expected number of members with participation index greater than or equal to z, whereas for an observed data set it is the actual number of such members.

The value of $u(z)$, for a certain specified value of z (say, $z = 0.7$) can itself serve as a quantitative measure of democracy in the decision-making process of a group.

Unlike the democracy index, which represents an average level of participation, $u(z)$ measures the number of highly influential members whose participation index exceeds a prescribed threshold. Thus, it provides complementary information about the distribution of participation within the group.

3. Results and Discussion

Using equations (8) and (9), respectively, the democracy index (D) and the percentage of fully participating members (x) have been calculated for different values of y_{max}, and their values are listed in Table 1.

Table 1. Calculated Values of the Democracy Index (D) and the Percentage of Fully Participating Members (x) for Different Values of y_{max}

y_{max}	D	x (%)
0.05	0.131304	0.000011
0.10	0.216080	0.049328

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0.20	0.378133	1.779026
0.30	0.476680	4.634625
0.40	0.532919	7.003644
0.50	0.567622	8.769738
0.60	0.590775	10.090264
0.70	0.607195	11.101141
0.80	0.619400	11.894787
0.90	0.628808	12.532272
0.95	0.632743	12.805811
1.00	0.636271	13.054533

Table 1 shows that both the democracy index (D) and the percentage of fully participating members (x) increase monotonically with y_{max}.

Based on equation (11), the values of $u(0.7)$, for $y_{max} = 0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9,\ 1.0$ are, respectively, 6.01, 6.56, 6.96, 7.25, 7.49 and 7.67.

A CASE STUDY

For a hypothetical 15-member group, I have constructed a data set based on practical experience and qualitative observation for the participation index (f_i) corresponding to 15 different members, each assigned an arbitrary serial number (i). The serial numbers serve only as identifiers and do not represent any ordering based on seniority, expertise, authority, experience, or level of participation. These values are listed in Table 2. The symbol f_i, referred to here as the participation index, is a quantitative measure of the perceived influence of the ith member in the decision-making process of the group, with $0 \leq f_i \leq 1$.

Table 2. Participation Index (f_i) Data for a Hypothetical 15-Member Group Based on Practical Experience and Qualitative Observation

Member Identifier (i) (integers from 1 to 15)	Participation Index (f_i) (A measure of the ith member's perceived influence in the decision-making process)
1	0.7
2	1.0
3	0.5
4	0.9
5	0.4
6	0.8
7	0.6
8	0.3
9	1.0
10	0.7
11	0.5
12	0.1
13	0.9
14	0.8
15	0.6

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The participation indices listed in Table 2 were constructed as a hypothetical but realistic data set based on practical experience and qualitative observation of group decision-making. The data are intended solely to illustrate the application of the proposed model to a hypothetical finite group. Although the data were not generated directly from equation (7), they were chosen to represent a plausible participation pattern for a small group and to examine whether the proposed theoretical model provides a reasonable description of such data.

Based on the data in Table 2, the democracy index (D) of the hypothetical group can be calculated directly from equation (1). For $m = 15$,

$$D_{Table-2} = \frac{1}{15}\sum_{i=1}^{m=15} f_i = \frac{9.9}{15} \approx 0.66$$

Among the 15 members, 2 members have $f_i = 1.0$. Therefore, the percentage of fully participating members is, $x = \frac{2}{15} \times 100 \approx 13.33\%$.

The maximum observed frequency is 2 and occurs at several participation levels between $y = 0.5$ and $y = 1.0$. This frequency distribution is broadly consistent with the theoretical prediction of the proposed model for $y_{max} \approx 1.0$.

These results are reasonably consistent with the theoretical predictions of the proposed model, as summarized in Table 1.

The values of $D = 0.66$, $x = 13.33\%$, and $u(0.7) = 8$ obtained from Table 2 are reasonably close to the corresponding theoretical predictions of the proposed model for $y_{max} = 1.0$, where $D = 0.6363$, $x = 13.05\%$, and $u(0.7) = 7.67$. The small differences are expected because Table 2 represents a finite group of only 15 members with participation indices restricted to discrete values, whereas the theoretical results are obtained from the analytical distribution given by equation (7). Thus, the hypothetical data provide an illustrative example that is reasonably consistent with the proposed mathematical model.

The proposed model has several strengths. It introduces a simple mathematical framework for quantifying the degree of democracy in the decision-making process of a group using only the participation indices (f_i) of its members. The model is easy to implement, requires only one empirical parameter (y_{max}), and provides complementary quantitative measures, including the democracy index (D), the percentage of fully participating members (x), and the number of members whose participation exceeds a specified threshold, $u(z)$. This makes the model computationally simple and potentially suitable for practical applications involving groups of different sizes. The monotonic increase of both D and x with y_{max} is also consistent with the intuitive expectation that greater participation should correspond to a higher degree of democracy.

The model also has certain limitations. The empirical distribution adopted for $k(y)$ is only one possible choice, and alternative functional forms may provide a better description of participation patterns in different types of groups. Moreover, because the adopted exponential distribution assigns a non-zero value to every participation level below full participation, the model cannot predict the ideal limiting case of complete democracy ($D = 1$ and $x = 100$). Thus, the present formulation should be regarded as a preliminary model that captures the general trend of democratic participation rather than an exact representation of all possible decision-making systems. Future work may explore alternative probability

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distributions that better represent participation patterns and satisfy the desired limiting behaviour while retaining the simplicity of the present formulation.

4. Concluding Remarks

A simple mathematical model has been proposed for quantifying the degree of democracy in the decision-making process of a group [6-8]. The model defines a democracy index based on the participation indices of individual members and introduces the percentage of fully participating members together with the number of members above a specified participation threshold as complementary measures. Using an empirical participation distribution, analytical expressions have been derived for these quantities, enabling their evaluation through a single empirical parameter.

The calculated results show that both the democracy index and the percentage of fully participating members increase monotonically with the parameter (y_{max}), which is consistent with the intuitive expectation that greater participation corresponds to a higher degree of democracy.

The proposed formulation has the advantages of simplicity, computational efficiency and ease of implementation. At the same time, the adopted empirical participation distribution represents only one possible choice and does not reproduce the ideal limiting case of complete democracy. Consequently, the present work should be regarded as a preliminary mathematical framework rather than a complete theory of democratic decision making.

Future experimental or survey-based studies may investigate alternative participation distributions, incorporate real survey data for parameter estimation, and examine the applicability of the proposed index to governments, organizations, academic bodies, corporate boards and other decision-making groups. Such developments may complement existing quantitative approaches to democracy measurement and facilitate comparisons across different decision-making contexts [3–5, 9, 10].

References

1. R. A. Dahl, *On Democracy*. New Haven, CT, USA: Yale University Press, 1998.

2. R. A. Dahl, *Polyarchy: Participation and Opposition*. New Haven, CT, USA: Yale University Press, 1971.

3. Economist Intelligence Unit, *Democracy Index 2024: What's Wrong with Representative Democracy?* London, U.K.: Economist Intelligence Unit, 2025.

4. M. G. Marshall and T. R. Gurr, *Polity IV Project: Political Regime Characteristics and Transitions, 1800–2018*. Vienna, VA, USA: Center for Systemic Peace, 2020.

5. S. I. Lindberg *et al.*, "V-Dem: A new way to measure democracy," *Journal of Democracy*, vol. 25, no. 3, pp. 159–169, 2014.

6. K. J. Arrow, *Social Choice and Individual Values*, 2nd ed. New Haven, CT, USA: Yale University Press, 1963.

7. A. Sen, *Collective Choice and Social Welfare*. San Francisco, CA, USA: Holden-Day, 1970.

8. D. S. Felsenthal and M. Machover, *The Measurement of Voting Power: Theory and Practice, Problems and Paradoxes*. Cheltenham, U.K.: Edward Elgar, 1998.

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9. G. L. Munck and J. Verkuilen, "Conceptualizing and Measuring Democracy: Evaluating Alternative Indices," *Comparative Political Studies*, vol. 35, no. 1, pp. 5–34, 2002.

10. M. Coppedge, J. Gerring, C. H. Knutsen, S. I. Lindberg, *et al.*, *V-Dem Country-Year Dataset v15*. Varieties of Democracy (V-Dem) Project, University of Gothenburg, Gothenburg, Sweden, 2025. DOI: 10.23696/vdemds25.



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