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Proposed NKIKE Distribution: Development and Implications for Population Growth Control

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Abstract

This paper develops a probability distribution from the migration-modified stochastic logistic growth model (MM-SLGM) to analyse the long-term behaviour of population dynamics under demographic uncertainty. By incorporating net migration and stochastic perturbations into the classical logistic model, it derived a stationary probability distribution called the NKIKE distribution that captures the randomness inherent in real-world population growth, particularly in developing economies. The study explored the properties of the developed model, its parameters and simulated its applicability. The resulting distribution, named the NKIKE distribution, provides critical insights into probable population outcomes, thresholds and the effectiveness of policy interventions. This approach bridges theoretical modelling and practical population control strategies, offering a flexible and robust tool for demographic planning amid socioeconomic and environmental fluctuations.

Keywords: NKIKE Distribution, Net-Migration, Population Control, Population Dynamics. Stationary Distribution, Stochastic Logistic Model

Introduction

Modification of the classical logistic growth model proposed by Verhulst (1838) has taken various forms over the last 170 years because of its assumption that population grows according to a sigmoidal curve bounded by carrying capacity, K, which represents the maximum sustainable population in given environments. However, the model ignores random events that cause population fluctuation. In the real

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world, where birth rates, death rates, migration and access to resources vary unpredictably, this assumption is unrealistic. Stochasticity was introduced through the Ito calculus and Wiener processes (i.e., Brownian motion), which transforms the model into a stochastic differential equation (SDE) which accommodates demographic events, capturing more realistic dynamics (Jean-Sebastien, Vallois, & Wantz-Mezieres, 2015; Shah, 2014; Allen, 2010; Higham, 2001). The stochastic model is indeed more appropriate for volatile environments where socioeconomic and health factors and policy interventions interact under uncertainty. Marrec, Bank, & Bertrand (2023) showed that some moment-closure approximations using stochastic growth dynamics describe the growth dynamics better than the deterministic prediction. Ejakpovi & Siloko (2023) modified the logistic growth model by incorporating an extra optimum stock as a constraint to fish population growth. Campillo, Joannides, & Larramendy-Valverde (2013) estimated the parameters of the finite difference numerical approximations and Monte Carlo approximations and likelihood estimations. Roy, Ghosh, & Saha (2022) extended the SLGM by proposing a maximum sustainable stable cell density solving the estimating the parameters of its probability density.

Population growth is a fundamental determinant of socioeconomic development, public health dynamics, resource allocation and environmental sustainability (Pang et al., 2024; Musa & Daradara, 2023; UN DESA, 2021; Opeoluwa, 2021). Across both the developed and developing economies, understanding the forces driving demographic transitions and the mathematical frameworks that describe them has become essential. Among these frameworks, growth models have extensively played a critical role in forecasting population change, analysing fertility-mortality-migration interactions and evaluating policy impacts. The classical deterministic models like the Malthusian and Verhulst logistic growth equations often fall short of encapsulating the randomness and real-life uncertainties inherent in demographic systems. This has led to the emergence and popularity of stochastic models, especially the stochastic logistic growth model (SLGM), which introduces random disturbances into the growth dynamics (Allen, 2003; Gardiner, 2009; Shah, 2014; Marrec, Bank, & Bertrand, 2023).

Understanding the probability distribution of the long-term behaviour of the stochastic model is essential. The probability density function (PDF) derived from the Fokker-Planck equation (FPE), also known as the forward Kolmogorov equation (FKE), describes how likely the population will be at a particular state over time (Risken, 1996). For some modified (extended) models, the steady-state (stationary) distribution reveals the probabilistic equilibrium of the population size. The FPE comprise two components: the deterministic drift term and the diffusion term, which, in steady-state or stationary conditions, create a valve for conversion to a probability distribution (Li, Tang, & Zhang, 2025; Precup & Rubbioni, 2022; Balakrishnam, 2021; Plastino, Wedemann, & Tsallis, 2021; Torregrossa & Toscani, 2018; Adesina, 2008). Shah (2014) developed an Erlang distribution from the stochastic logistic distribution and evaluated its mean-variance in a steady state.

Migration, often called the “third pillar” of population change alongside fertility and mortality, exerts complex influences on population structure and growth. Net migration, which accounts for the balance between immigration and emigration, can either amplify or suppress population expansion depending on its magnitude, direction and drivers (Weeks, 2021). In many developing countries, especially in sub-Saharan Africa in general and Nigeria in particular, rural-urban migration, international labour mobility, especially the recent “Japa’ syndrome and displacement due to conflict or climate change contribute to rapid urbanisation and uneven demographic pressure (UN DESA, 2022). However, despite its growing relevance, migration is often treated as exogenous or omitted altogether in traditional growth models. This omission weakens the models’ ability to predict real-world outcomes where migration acts as a demographic “shock absorber” or amplifier. Consequently, incorporating migration as an endogenous component of stochastic population models enables a more nuanced and realistic representation of

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demographic dynamics (Preston, Heuveline & Guillot, 2001; Rice & Papadopoulos, 2009; Entwisle, Verdery, & Williams, 2021; Watson, Lenton, & de Campos, 2023). Ikegwu & Nkemnole (2025) proposed a migration-modified stochastic logistic growth model by extending the classical stochastic growth model with an additive net-migration component.

One critical motivation for deriving the stationary probability distribution from the MM-SLGM is its implication for population control strategies. In regions experiencing explosive population growth, like in sub-Saharan Africa, accompanied by poor health infrastructure, inadequate education systems, and limited employment opportunities, understanding the dynamics and controllability of population growth is imperative (Bongaarts, 2016; Gotmark & Andersson, 2023). The model development helps simulate various scenarios under different values of migration rate, volatility and intrinsic growth rate.

This paper addresses a critical gap in demographic modelling by developing a probability distribution function (PDF) from the Ikegwu & Nkemnole (2025) **Migration-Modified Stochastic Logistic Growth Model (MM-SLGM)** named the "**NKIKE**" distribution, uncovering deeper probabilistic insight into population outcomes under migration pressure and demographic shocks. This study derives the probability density function of the proposed "**NKIKE**" distribution, understands the model's structure and contextual relevance, validates it through simulations and draws practical conclusions for population growth control strategies, particularly in developing economies where demographic volatility is high.

2.0 Methodology

The study proposed a new demographic probability distribution function of the Ikegwu & Nkemnole (2025) **Migration Modified Logistic Growth Model (MM-SLGM)** given below:

$$dP = \left[rP\left(1 - \frac{P}{K}\right) + mP\right]dt + fPdWt \quad (1)$$

Where

P is the population size

K is the carrying capacity

m is the net migration rate

f is the stochastic coefficient

dWt is the Wiener's random error

This is a stochastic logistic growth model with:

$rP\left(1 - \frac{P}{K}\right)$ as the intrinsic logistic growth model (Verhulst, 1838)

mP is an additive deterministic migration term (net migration rate per term)

$fPdWt$ is the multiplicative environmental stochasticity (via Brownian motion)

Proposing a probability distribution function named the "NKIKE" distribution

Rewriting the SDE in equation (1) into:

$$dP = \mu(P)dt + \sigma(P)dWt \quad (2)$$

Where

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The drift term $\mu(P) = (r + m)P - \frac{r}{K}P^2$ and

The diffusion term is $\sigma(P) = fP$ and $\sigma^2(P) = f^2P^2$

The Fokker-Planck Equations (FPE)

The Fokker-Planck equation for the pdf p(P, t) is given as:

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial P}[\mu(P)p] + \frac{1}{2}\frac{\partial^2}{\partial P^2}\left[\sigma^2(P)p\right] \quad (3)$$

At stationarity, $\frac{\partial p}{\partial t} = 0$; hence,

$$\frac{dp}{dt} = \frac{d}{dP}\left[\mu(P)p_{st}(P)\right] + \frac{1}{2}\frac{d^2}{dP^2}\left[\sigma^2(P)p_{st}(P)\right] = 0 \quad (4)$$

Solving this gives the stationary distribution $p_{st}(P)$

General Stationary Solution Form

The stationary solution of a one-dimensional diffusion process according to Gardiner (2009) gives the stationary probability distribution. $p_{st}(P)$ is given as:

$$p_{st}(P) = \frac{C}{\sigma^2(P)}.\ exp\left(\int \frac{2\mu(P)}{\sigma^2(P)}dP\right) \quad (5)$$

Substituting for the drift and diffusion terms

$$p_{st}(P) = \frac{C}{f^2P^2}.\ exp\left(\int \frac{2\left[(r+m)P - \frac{r}{K}P^2\right]}{f^2P^2}dP\right) \quad (6)$$

Simplifying the Integral

To simplify the integral in equation (6) in the exponent:

$$\int \frac{2\left[(r+m)P - \frac{r}{K}P^2\right]}{f^2P^2}dP = \frac{2}{f^2}\int\left(\frac{(r+m)}{P} - \frac{r}{K}\right)dP \quad (7)$$

Solving the integral

$$\int\left(\frac{(r+m)}{P} - \frac{r}{K}\right)dP = (r + m)\ln ln\, P - \frac{r}{K}P \quad (8)$$

Therefore, putting all the parts together for the model in eqn. (6),

$$p_{st}(P) = \frac{C}{f^2P^2}.\ exp\left(\frac{2}{f^2}\left[(r + m)\ln ln\, P - \frac{r}{K}P\right]\right) \quad (9)$$

$$= \frac{C}{f^2}.\ P^{\frac{2(r+m)}{f^2}-2}\ exp\left(-\frac{2r}{f^2K}P\right) \quad (10)$$

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Equation (10) is the probability density function with C as the normalising constant such that $\int_0^\infty p_{st}(P)dP = 1$.

Deriving the normalising constant C

Recall eqn. (10), let's define the following

$$\alpha = P^{\frac{2(r+m)}{f^2}-2}$$

$$\beta = \frac{2r}{f^2 K}$$

Then,

$$p_{st}(P) = \frac{C}{f^2}. P^{\alpha-2} e^{-\beta P} \quad (11)$$

But for the probability density function (pdf) in equation (11) to satisfy the condition.

$$\int_0^\infty p_{st}(P)dP = 1 \quad (12)$$

Compute, therefore:

$$\int_0^\infty \frac{C}{f^2}. P^{\alpha-2} e^{-\beta P} dP = 1 \quad (13)$$

From the standard gamma integral identity:

$$\int_0^\infty x^{s-1} e^{-\gamma x} dP = \frac{\Gamma(s)}{\gamma^s}, \qquad for\ s > 0, \gamma > 0 \quad (14)$$

Letting

$$x = p, s = \alpha - 1 = \frac{2(r+m)}{f^2} - 1, \gamma = \beta = \frac{2r}{f^2 K}$$

Then:

$$\int_0^\infty x^{\alpha-2} e^{-\beta x} dP = \frac{\Gamma(\alpha-1)}{\beta^{\alpha-1}},$$

Solving for the normalising constant C

Substituting back into the normalisation equation (11):

$$\frac{C}{f^2}. \frac{\Gamma(\alpha-1)}{\beta^{\alpha-1}} = 1$$

Solving for C

$$C = f^2 \frac{\beta^{\alpha-1}}{\Gamma(\alpha-1)} \quad (15)$$

Plugging back into the normalised constant

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$$C = f^2 \frac{1}{\Gamma\left(\frac{2(r+m)}{f^2}-1\right)} \cdot \left(\frac{2r}{f^2K}\right)^{\frac{2(r+m)}{f^2}-1} \quad (16)$$

The full final normalised probability density function with the normalised constant C is substituted into the original pdf in eqn. (10)

$$p_{st}(P) = \frac{C}{f^2} \cdot P^{\frac{2(r+m)}{f^2}-2} exp\left(-\frac{2r}{f^2K}P\right)$$

Replacing C above with eqn. (14)

$$p_{st}(P) = \left[\frac{1}{\Gamma\left(\frac{2(r+m)}{f^2}-1\right)} \cdot \left(\frac{2r}{f^2K}\right)^{\frac{2(r+m)}{f^2}-1}\right] \cdot P^{\frac{2(r+m)}{f^2}-2} exp\left(-\frac{2r}{f^2K}P\right), \quad (17)$$

$$f > 0,\ r > 0,\ m > 0,\ P > 0,\ K > 0$$

Equation (17) is the full and final probability density function called the **NKIKE** distribution from the Migration Modified Stochastic Logistic Growth Model (MM-SLGM).

The form of the pdf resembles the Gamma distribution with four parameters.

$$Gamma(x;\ \gamma,\ s) = \frac{1}{\Gamma(s)\gamma^s} x^{s-1}\, e^{-\gamma x} \quad (18)$$

With $s = \alpha - 1 = \frac{2(r+m)}{f^2} - 1$ and rate $\beta = \frac{2r}{f^2K}$

The normalising constant ensures that the total probability across the domain $= P \in (0, \infty)$. Also, well-defined conditions for the $\Gamma(\alpha - 1)$ to exist and be finite, it is required that

$$\alpha - 1 = \frac{2(r+m)}{f^2} - 1 > 0 \Rightarrow r + m > \frac{f^2}{2} \quad (19)$$

This ensures that the PDF is integrable and the population does not vanish due to noise or low growth/ migration.

Testing that the PDF is Proper

To show that the PDF is proper,

From equation (17), the proposed NKIKE probability density function is

$$p_{st}(P) = \left[\frac{1}{\Gamma\left(\frac{2(r+m)}{f^2}-1\right)} \cdot \left(\frac{2r}{f^2K}\right)^{\frac{2(r+m)}{f^2}-1}\right] \cdot P^{\frac{2(r+m)}{f^2}-2} exp\left(-\frac{2r}{f^2K}P\right),$$

$$f > 0,\ r > 0,\ m > 0,\ P > 0,\ K > 0$$

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The probability density function p(P) is proper if:

a. It is non-negative for all P > 0, i.e., $p_{st}(P)\ \forall\ P \in (0,\ \infty)$

b. It integrates to 1 over the domain, i.e., $\int_0^\infty p_{st}(P)dP = 1$

To show the integrability of the model,

$$\int_0^\infty \left[\frac{(\beta)^{\alpha-1}}{\Gamma(\alpha-1)}\right]. P^{\alpha-2} e^{-\beta P} dP = 1 \quad (20)$$

Where,

$$\alpha = \frac{2(r+m)}{f^2}, \beta = \frac{2r}{f^2 K}$$

$$P^{\alpha-2} > 0 \ for\ P > 0$$

$$f^2 > 0$$

$$e^{-\beta P} > 0 \ for\ P > 0$$

Therefore,

$$p_{st}(P) \geq 0, \ for\ all\ P > 0$$

Prove the convergence of the Integral

To show that:

$$\int_0^\infty P^{\alpha-2} e^{-\beta P} dP < \infty \quad (21)$$

Having obtained C in eqn. (17) as

$$C = f^2 \frac{\beta^{\alpha-1}}{\Gamma(\alpha-1)}$$

And

$$\int_0^\infty p_{st}(P)dP = 1$$

Then,

$$\int_0^\infty P^{\alpha-2} e^{-\beta P} dP = \frac{\Gamma(\alpha-1)}{f^2 \beta^{\alpha-1}}$$

Therefore,

$$\int_0^\infty \left[\frac{(\beta)^{\alpha-1}}{\Gamma(\alpha-1)}\right]. P^{\alpha-2} e^{-\beta P} dP = \left[\frac{(\beta)^{\alpha-1}}{\Gamma(\alpha-1)}\right] \int_0^\infty P^{\alpha-2} e^{-\beta P} dP$$

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$$= \left[\frac{\beta^{\alpha-1}}{\Gamma(\alpha-1)}\right].\ \frac{\Gamma(\alpha-1)}{\beta^{\alpha-1}} = 1 \tag{22}$$

Hence, equation (17) is a proper probability distribution function.

Plugging back $\Gamma(\alpha - 1)$ and $\beta^{\alpha-1}$, the NKIKE distribution as in eqn. (17)

$$p_{st}(P) = \left[\frac{1}{\Gamma\left(\frac{2(r+m)}{f^2}-1\right)}.\left(\frac{2r}{f^2K}\right)^{\frac{2(r+m)}{f^2}-1}\right].\ P^{\frac{2(r+m)}{f^2}-2}\ exp\left(-\frac{2r}{f^2K}P\right),$$

$$f > 0,\ r > 0,\ m > 0,\ P > 0,\ K > 0$$

is a proper PDF with $r + m > \frac{f^2}{2}$. This model yields a well-defined, normalised, non-negative probability distribution for population sizes under the migration-modified stochastic logistic growth model (MM-SLGM).

Properties of the Proposed NKIKE Distribution

The properties explored include the expectation, variance, moments, skewness, kurtosis, mode, median, quartiles and percentiles of the distribution.

Estimating the Expectation and Variance of the PDF.

The stationary PDF is given as

$$p_{st}(P) = \left[\frac{(\beta)^{\alpha-1}}{\Gamma(\alpha-1)}\right].\ P^{\alpha-2}\ e^{-\beta P} \tag{23}$$

Where

$$\alpha = \frac{2(r+m)}{f^2}, \text{ and } \beta = \frac{2r}{f^2K}$$

This is recognised as the Gamma type distribution of four parameters with the shape parameter $k = \alpha - 1 = \frac{2(r+m)}{f^2} - 1$, the rate parameter $\beta = \frac{2r}{f^2K}$ and the scale parameter $\theta = \frac{1}{\beta} = \frac{f^2K}{2r}$

The Expected Value (Mean) of the Proposed NKIKE Distribution

The expected value of the NKIKE distribution reduced to a known Gamma type distribution of four parameters with shape k and rate β is:

$$E[P] = \frac{k}{\beta} \tag{24}$$

Plugging in the shape and rate parameters,

$$E[P] = \frac{1}{\frac{2r}{f^2K}}.\frac{2(r+m)}{f^2} - 1 = \frac{2(r+m)-f^2}{2r}\ K \tag{25}$$

NB: As $f \to 0$, then the mean approaches the deterministic logistic equilibrium:

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$E[P] \rightarrow K \cdot \frac{r+m}{r}$. Also, a higher net migration rate m increases the expected population and a higher diffusion intensity. f^2 reduces the expected value due to stronger environmental variability.

The Variance of the PDF

The variance of the NKIKE distribution reduced to a known Gamma type distribution of four parameters with shape k and rate β is

$$Var[P] = \frac{k}{\beta^2} = \left(\frac{2(r+m)}{f^2} - 1\right).\left(\frac{f^2K}{2r}\right)^2 \quad (26)$$

$$= \left(\frac{2(r+m)}{f^2} - 1\right).\frac{f^4K^2}{4r^2} = \frac{\left(2(r+m) - f^2\right).f^2.K^2}{4r^2}$$

Therefore, the variance of the pdf is

$$Var[P] = \frac{\left(2(r+m) - f^2\right).f^2.K^2}{4r^2} \quad (27)$$

The variance increases with higher environmental noise, f^2, implying more random fluctuations. If $r + m$ is large, the population is more resilient to noise and if $f^2 \geq 2(r + m)$, the variance becomes non-positive or undefined, indicating population instability or extinction risk.

Moments of the Proposed NKIKE Distribution

The nth raw moment of the NKIKE distribution reduced to a known Gamma type distribution of four parameters, $X \sim Gamma(k, \beta)$ is

$$\mu_n^{'} = E\left[X^n\right] = \frac{\Gamma(k+n)}{\beta^n\Gamma(k)} = \theta^n \frac{\Gamma(k+n)}{\Gamma(k)} \quad (30)$$

Where

$\mu_n^{'}$ is the nth moment about the origin

$\theta = \frac{1}{\beta}$ is the scale parameter

$\Gamma(.)$ Is the gamma function

Compute the First Four Moments

Let $k = \alpha - 1 = \frac{2(r+m)}{f^2} - 1$, $\beta = \frac{2r}{f^2K}$, and $\theta = \frac{1}{\beta} = \frac{f^2K}{2r}$

First Moment (Mean)

The first moment of the proposed distribution is given as

$$\mu_1^{'} = E[P] = \frac{k}{\beta} = k.\theta = \frac{2(r+m) - f^2}{2r} K \quad (31)$$

The expected population increases with net migration, m, decreases with stochasticity, f^2, and grows proportionally with carrying capacity.

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Second Moment

The second moment of the proposed distribution is given as

$$\mu_2^{'} = E\left[P^2\right] = \frac{\Gamma(k+2)}{\beta^2\Gamma(k)} = \frac{k(k+1)}{\beta^2} \quad (32)$$

$$\therefore \mu_2^{'} = E\left[P^2\right] = \left(\frac{2(r+m)}{f^2} - 1\right)\left(\frac{2(r+m)}{f^2}\right)\left(\frac{f^4K^2}{4r^2}\right) \quad (33)$$

This reflects the spread or fluctuation in the squared population size and is important in computing the variance.

Third Moment

The third moment of the proposed distribution is given as

$$\mu_3^{'} = E\left[P^3\right] = \frac{\Gamma(k+3)}{\beta^3\Gamma(k)} = \frac{k.(k+1)(k+2)}{\beta^3} \quad (34)$$

$$\mu_3^{'} = E\left[P^3\right] = \left(\frac{2(r+m)}{f^2} - 1\right)\left(\frac{2(r+m)}{f^2}\right)\left(\frac{2(r+m)}{f^2} + 1\right)\left(\frac{f^6K^3}{8r^3}\right) \quad (35)$$

This affects the skewness (asymmetry) of the distribution. If skewness > 0, it implies a right-skewed population distribution.

Fourth Moment

The fourth moment of the proposed distribution is given as

$$\mu_4^{'} = E\left[P^4\right] = \frac{\Gamma(k+4)}{\beta^4\Gamma(k)} = \frac{k.(k+1)(k+2)(k+3)}{\beta^4} \quad (36)$$

$$\mu_4^{'} = E\left[P^4\right] = \left(\frac{2(r+m)}{f^2} - 1\right)\left(\frac{2(r+m)}{f^2}\right)\left(\frac{2(r+m)}{f^2} + 1\right)\left(\frac{2(r+m)}{f^2} + 2\right)\left(\frac{f^8K^4}{16r^4}\right) \quad (37)$$

Central Moments of the Proposed NKIKE Distribution

Variance

The variance obtained using moments for the proposed distribution is given as

$$Var[P] = \mu_2 - \mu_1^2 = \frac{k}{\beta^2} \quad (38)$$

$$\therefore Var[P] = \frac{\left(2(r+m) - f^2\right).f^2.K^2}{4r^2} \quad (39)$$

Skewness of the Proposed NKIKE distribution

The skewness of the proposed distribution is given as

$$\gamma_1 = \frac{\mu_3 - 3\mu_1\mu_2 + 3\mu_1^3}{\sigma^3} = \frac{2}{\sqrt{k}} = \frac{2}{\sqrt{\left(\frac{2(r+m)}{f^2} - 1\right)}} \quad (40)$$

$$\therefore \gamma_1 = \frac{2}{\sqrt{\left(\frac{2(r+m)}{f^2} - 1\right)}} \quad (41)$$

The distribution is always positive, confirming that the PDF is right-skewed.

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Kurtosis of the Proposed NKIKE distribution

The Kurtosis of the proposed distribution is given as

$$\gamma_2 = \frac{\mu_4 - 4\mu_3\mu_1 + 6\mu_2\mu_1^2 - 3\mu_1^4}{\sigma^4} = \frac{6}{k} + 3 \quad (42)$$

$$\therefore \gamma_2 = \frac{6}{\left(\frac{2(r+m)}{f^2} - 1\right)} + 3 \quad (43)$$

The distribution is greater than 3 (normal distribution); hence, the distribution is leptokurtic (peaked with heavy tails).

The Mode of the Proposed NKIKE Distribution

The mode (most likely value) of the proposed distribution with k > 1 is given as

$$Mode(P) = \frac{k-1}{\beta} = \left(\frac{2(r+m)}{f^2} - 2\right).\frac{f^2K}{2r} \quad (44)$$

$$= \frac{2(r+m) - 2f^2}{2r}.K \quad (45)$$

The mode is slightly less than the mean when the shape parameter, k > 2, due to right skewness, increased noise, f^2 lowers the most likely population size, and larger net migration m raises the mode.

The Median of the Proposed NKIKE distribution

The median $P_{0.5}$ of the proposed distribution does not have a closed-form formula but can be approximated or found numerically and is given as

$$Median(P) = k.\theta.\left(1 - \frac{1}{3k}\right) \quad (46)$$

$$= \left(\frac{2(r+m)}{f^2} - 1\right).\frac{f^2K}{2r}.\left(1 - \frac{1}{3\left(\frac{2(r+m)}{f^2} - 1\right)}\right) \quad (47)$$

This is only an approximation. For accurate results, we solve

$$\int_0^{P_{0.5}} p_{st}(P)dP = 0.5 \quad (48)$$

Which requires numerical integration (e.g., in R).

The median is always less than the mean for a right-skewed distribution; higher stochasticity flattens the curve and shifts the median leftward, and the median is especially relevant for policy thresholds (e.g., 50% of the population below certain levels).

Percentiles of the MM-SGLM

The general formula for computing the p^{th} percentile P_p is given as:

$$F\left(P_p\right) = \int_0^{P_p} p_{st}(P)dP = p \quad (49)$$

For Gamma distributions, the cumulative distribution function

$$F(P; k, \beta) = \frac{\gamma(k, \beta P)}{\Gamma(k)} \quad (50)$$

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Where,

$\gamma(k,\ \beta P)$ is the lower incomplete gamma function

$\Gamma(k)$ is the gamma function

Thus, the percentile P_p satisfies the following.

$$\frac{\gamma\left(k,\ \beta P_p\right)}{\Gamma(k)} = p \Rightarrow \frac{1}{\beta}\gamma^{-1}(k,\ p.\Gamma(k)) \quad (51)$$

Approximating the quartiles has no simple closed form; numerical approximation or software is used (e.g., R or Python or Excel)

The first quartile (Q_1) is equivalent to $P_{0.25,}$ while the third quartile (Q_3) is equivalent to $P_{0.75}$.

These can be computed numerically as:

$$P_{0.25} \approx qgamma(0.25,\ k,\ \theta),\ \ P_{0.75} \approx qgamma(0.75,\ k,\ \theta) \quad (52)$$

Where qgamma is the inverse gamma CDF.

Percentiles help assess thresholds for policy, interventions and inequality studies.

Estimating the Parameters of the Proposed NKIKE Distribution

To derive all the parameters of the proposed NKIKE distribution from the stochastic logistic growth model with net migration using the Maximum Likelihood Estimation (MLE) method, approach it systematically. From the stationary probability density function (equation 17), which follows a Gamma type distribution of four parameters, the estimate of its parameters using MLE based on observed data is given.

$$p(P; r,\ m,\ f,\ K) = \frac{\beta^k}{\Gamma(k)}.\ P^{k-1}\ e^{-\beta P},\quad P > 0$$

This is recognised as the Gamma distribution:

$$(Gamma(P; k,\ \beta) \qquad with\ parameters$$

$$\text{shape } k = \frac{2(r+m)}{f^2} - 1, \text{ rate } \beta = \frac{2r}{f^2 K}, \text{ scale } \theta = \frac{1}{\beta} = \frac{f^2 K}{2r}$$

Maximum Likelihood Estimator (MLE) Assumption and Setup

Let P_1, P_2, ..., P_n be an independent and identically distributed (i.i.d.) sample from the stationary population distribution. The estimate of the Gamma parameters k and β.

$$L(k,\ \beta) = \prod_{i=1}^{n} \frac{\beta^k}{\Gamma(k)}.\ P_i^{\,k-1}\ e^{-\beta P_i} \quad (53)$$

The log-likelihood function becomes

$$ln(L(k,\ \beta) = \sum_{i=1}^{n}\left[k ln\beta - ln\Gamma(k) + (k-1) ln P_i - \beta P_i\right] \quad (54)$$

Simplifying eqn. (54)

$$ln(L(k,\ \beta) = nk ln\beta - n ln\Gamma(k) + (k-1) ln \sum_{i=1}^{n} P_i - \beta \sum_{i=1}^{n} P_i \quad (55)$$

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Derive MLE for β and k

Differentiate eqn. (55) w.r.t β and set the derivative to 0

$$\frac{\partial L}{\partial \beta} = \frac{nk}{\beta} - \sum_{i=1}^{n} P_i = 0 \Rightarrow \hat{\beta} = \frac{nk}{\sum_{i=1}^{n} P_i} = \frac{k}{\bar{P}} \quad (56)$$

$$= \frac{1}{\bar{P}} \cdot \frac{2(r+m)}{f^2} - 1 = \frac{2(r+m) - f^2}{\bar{P} f^2} \quad (57)$$

Differentiate eqn. (55) w.r.t k and set the derivative to 0

$$\frac{\partial L}{\partial \beta} = n ln\beta - n\psi(k) + \sum_{i=1}^{n} lnP_i = 0 \quad (58)$$

Where $\psi(k) = \frac{d}{dk} ln\Gamma(k)$ is the digamma function.

$$n ln\beta - n\psi(k) + \sum_{i=1}^{n} lnP_i = 0 \Rightarrow ln\hat{\beta} = \psi\left(\hat{k}\right) - \overline{lnP} \quad (59)$$

Where $\overline{lnP} = \frac{1}{n} \sum lnP_i$

Once $\hat{k}$ and $\hat{\beta}$ are estimated,

From the shape parameter,

$$\hat{k} = \frac{2\left(\hat{r}+\hat{m}\right)}{\hat{f}^2} - 1 \Rightarrow \hat{r} + \hat{m} = \frac{\hat{f}^2}{2}\left(\hat{k} + 1\right) \quad (60)$$

From the rate parameter,

$$\hat{\beta} = \frac{2\hat{r}}{\hat{f}^2 \hat{K}} \Rightarrow \hat{r} = \frac{\hat{\beta} \hat{f}^2 \hat{K}}{2} \quad (61)$$

These expressions can be combined to solve for the original biological parameters. $\hat{r}$, $\hat{m}$, $\hat{f}^2$ and $\hat{K}$, if sufficient degrees of freedom or known quantities are available (fixing K from data).

The model was simulated using code written in an R environment.

Results

The simulated results of the proposed NKIKE distribution are presented in this section.

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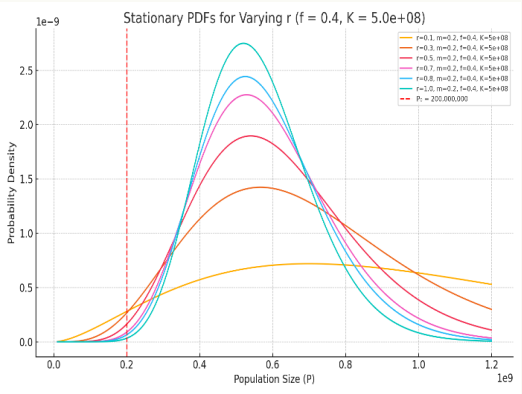


Fig. 1: The probability plot of the NKIKE distribution at f= 0.4, m = 0 at different r.

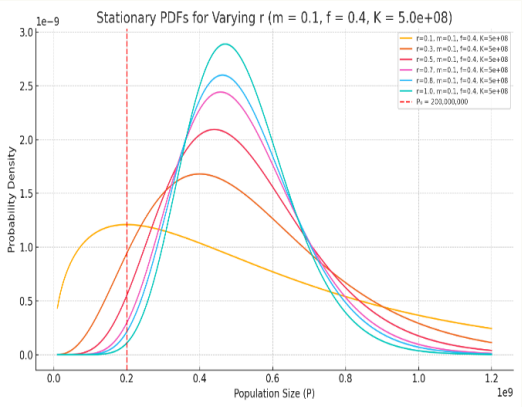


Fig. 2: The probability plot of the NKIKE distribution at f= 0.4, m = 0.1 and at different r.

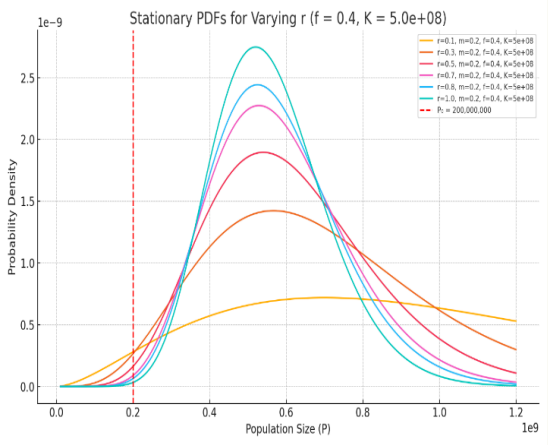


Fig. 3: The probability plot of the NKIKE distribution At f = 0.4, m = 0.2 and at different r.As r increases, the peakedness of the model increases.

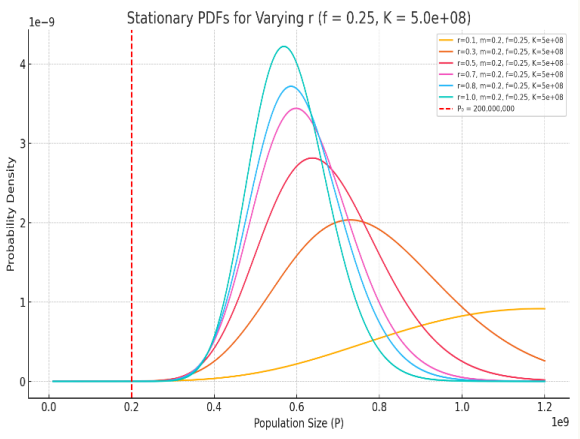


Fig. 4: The probability plot of the NKIKE distribution at f= 0.4, m = 0.25 at different r.

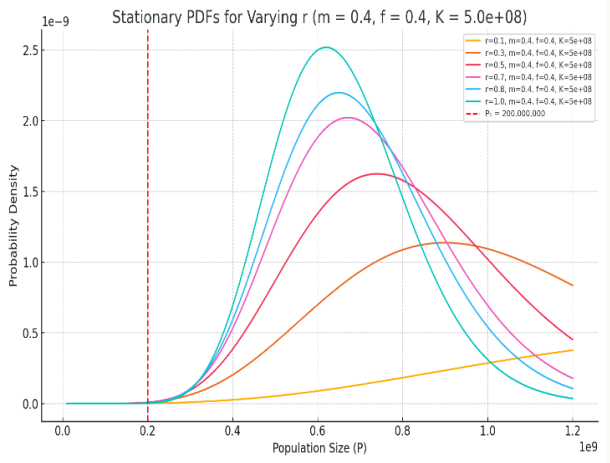


Fig. 5: The probability plot of the NKIKE distribution at f= 0.4, m = 0.4 and at different r.

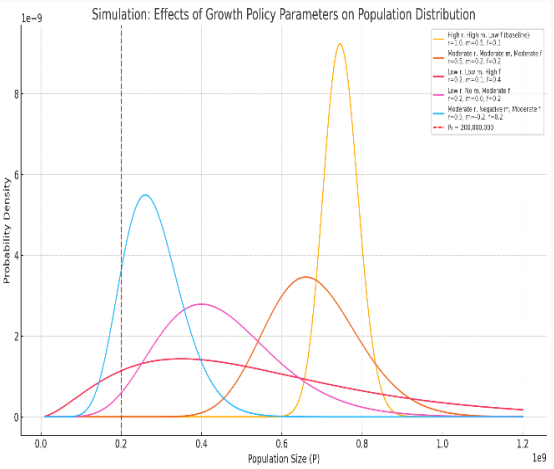


Fig.6: Effects of growth policy parameters on NKIKE distribution

High migration and low noise may lead to stable population growth around the carrying capacity; increased stochasticity (due to environmental shocks or policy failures) may widen the distribution, suggesting instability or unpredictability, and negative net migration (emigration) may help stabilize populations around sustainable levels. Hence, the probabilistic view enables policymakers to design migration-sensitive interventions such as rural development programs to reduce urban pull or regional incentives to absorb surplus populations sustainably.

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Simulated Parameters of the NKIKE model (at K = 1.0e+09 and P = 2.0e+08)

Table 1: The parameters and properties of the NKIKE distribution (r = 0.1) and RMSE (at m= 0) (at K = 1.0e+09 and P = 2.0e+08)

R	m	F	k	β	θ	Mean	Variance	Skewness	Kurtosis	Mode	RMSE (m=0)
0.1	-0.05	0.1	9.0	2e-08	50000000.0	4.50e+8	2.25e+16	0.667	3.667	4.0e+08	500000000.0
0.1	-0.05	0.15	3.4444	1e-08	112500000.0	3.87e+8	4.36e+16	1.078	4.742	275000000.0	500000000.0
0.1	-0.05	0.2	1.5	0.0	200000000.0	3.00e+8	6.00e+16	1.633	7.0	100000000.0	500000000.0
0.1	-0.05	0.25	0.6	0.0	312500000.0	1.87e+8	5.89e+16	2.582	13.0	0.0	500000000.0
0.1	-0.05	0.3	0.1111	0.0	450000000.0	5.00 e+7	2.25e+16	6.0	57.0	0.0	500000000.0
0.1	0.0	0.1	19.0	2e-08	50000000.0	9.50 e+8	4.75e+16	0.459	3.316	900000000.0	0.0
0.1	0.0	0.15	7.8889	1e-08	112500000.0	8.87 e+8	9.98e+16	0.712	3.761	775000000.0	0.0
0.1	0.0	0.2	4.0	0.0	200000000.0	8.00 e+8	1.6e+17	1.0	4.5	600000000.0	0.0
0.1	0.0	0.25	2.2	0.0	312500000.0	6.87 e+8	2.15e+17	1.348	5.727	375000000.0	0.0
0.1	0.0	0.3	1.2222	0.0	450000000.0	5.50 e+8	2.47e+17	1.809	7.909	100000000.0	0.0
0.1	0.0	0.35	0.6327	0.0	612500000.0	3.87 e+8	2.37e+17	2.514	12.484	0.0	0.0
0.1	0.0	0.4	0.25	0.0	800000000.0	2.00 e+8	1.60e+17	4.0	27.0	0.0	0.0
0.1	0.05	0.1	29.0	2e-08	50000000.0	1.45 e+9	7.25e+16	0.371	3.207	1400000000.0	500000000.0
0.1	0.05	0.15	12.3333	1e-08	112500000.0	1.39 e+9	1.56e+17	0.569	3.486	1275000000.0	500000000.0
0.1	0.05	0.2	6.5	0.0	200000000.0	1.30 e+9	2.60e+17	0.784	3.923	1100000000.0	500000000.0
0.1	0.05	0.25	3.8	0.0	312500000.0	1.19 e+9	3.71e+17	1.026	4.579	875000000.0	500000000.0
0.1	0.05	0.3	2.3333	0.0	450000000.0	1.05 e+9	4.725e+17	1.309	5.571	600000000.0	500000000.0
0.1	0.05	0.35	1.449	0.0	612500000.0	8.87 e+8	5.44e+17	1.661	7.141	275000000.0	500000000.0

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0.1	0.05	0.4	0.875	0.0	800000000.0	7.00 e+8	5.60e+17	2.138	9.857	0.0	500000000.0
0.1	0.1	0.1	39.0	2e-08	50000000.0	1.95 e+9	9.75e+16	0.32	3.154	1900000000.0	1000000000.0
0.1	0.1	0.15	16.7778	1e-08	112500000.0	1,89 e+9	2.12e+17	0.488	3.358	1775000000.0	1000000000.0
0.1	0.1	0.2	9.0	0.0	200000000.0	1.80 e+9	3.60e+17	0.667	3.667	1600000000.0	1000000000.0
0.1	0.1	0.25	5.4	0.0	312500000.0	1.69 e+9	5.27e+17	0.861	4.111	1375000000.0	1000000000.0
0.1	0.1	0.3	3.4444	0.0	450000000.0	1.55 e+9	6.97e+17	1.078	4.742	1100000000.0	1000000000.0
0.1	0.1	0.35	2.2653	0.0	612500000.0	1.39 e+9	8.50e+17	1.329	5.649	775000000.0	1000000000.0
0.1	0.1	0.4	1.5	0.0	800000000.0	1.20 e+9	9.60e+17	1.633	7.0	400000000.0	1000000000.0
0.1	0.2	0.1	59.0	2e-08	50000000.0	2.95 e+9	1.47e+17	0.26	3.102	2900000000.0	2000000000.0
0.1	0.2	0.15	25.6667	1e-08	112500000.0	2.89 e+9	3.25e+17	0.395	3.234	2775000000.0	2000000000.0
0.1	0.2	0.2	14.0	0.0	200000000.0	2.80 e+9	5.60e+17	0.535	3.429	2600000000.0	2000000000.0
0.1	0.2	0.25	8.6	0.0	312500000.0	2.69 e+9	8.40e+17	0.682	3.698	2375000000.0	2000000000.0
0.1	0.2	0.3	5.6667	0.0	450000000.0	2.55 e+9	1.15e+18	0.84	4.059	2100000000.0	2000000000.0
0.1	0.2	0.35	3.898	0.0	612500000.0	2.39 e+9	1.46e+18	1.013	4.539	1775000000.0	2000000000.0
0.1	0.2	0.4	2.75	0.0	800000000.0	2.20 e+9	1.76e+18	1.206	5.182	1400000000.0	2000000000.0

NB: r = intrinsic growth rate, m = net migration rate, f is strength of multiplicative noise, K is carrying capacity, k = shape parameter, **β** = Rate parameter, **θ** = Scale parameter.

Table 2: The parameters and properties of the NKIKE distribution (r = 0.2) and RMSE (at m= 0)

r	m	F	k	B	θ	Mean	Variance	Skewness	Kurtosis	Mode	RMSE (m = 0)
0.2	-0.1	0.1	19.0	4e-08	25000000.0	475000000.0	1.19e+16	0.459	3.316	450000000.0	500000000.0
0.2	-0.1	0.15	7.89	2e-08	56250000.0	443750000.0	2.50e+16	0.712	3.761	387500000.0	500000000.0

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0.2	-0.1	0.2	4.0	1e-08	100000000.0	400000000.0	4.00e+16	1.0	4.5	300000000.0	500000000.0
0.2	-0.1	0.25	2.2	1e-08	156250000.0	343750000.0	5.37e+16	1.348	5.727	187500000.0	500000000.0
0.2	-0.1	0.3	1.22	0.0	225000000.0	275000000.0	6.19e+16	1.809	7.909	50000000.0	500000000.0
0.2	-0.1	0.35	0.63	0.0	306250000.0	193750000.0	5.93e+16	2.514	12.484	0.0	500000000.0
0.2	-0.1	0.4	0.25	0.0	400000000.0	100000000.0	4.00e+16	4.0	27.0	0.0	500000000.0
0.2	-0.05	0.1	29.0	4e-08	25000000.0	725000000.0	1.81e+16	0.371	3.207	700000000.0	250000000.0
0.2	-0.05	0.15	12.33	2e-08	56250000.0	693750000.0	3.90e+16	0.569	3.486	637500000.0	250000000.0
0.2	-0.05	0.2	6.5	1e-08	100000000.0	650000000.0	6.50e+16	0.784	3.923	550000000.0	250000000.0
0.2	-0.05	0.25	3.8	1e-08	156250000.0	593750000.0	9.28e+16	1.026	4.579	437500000.0	250000000.0
0.2	-0.05	0.3	2.33	0.0	225000000.0	525000000.0	1.18e+17	1.309	5.571	300000000.0	250000000.0
0.2	-0.05	0.35	1.45	0.0	306250000.0	443750000.0	1.36e+17	1.661	7.141	137500000.0	250000000.0
0.2	-0.05	0.4	0.87	0.0	400000000.0	350000000.0	1.40e+17	2.138	9.857	0.0	250000000.0
0.2	0.0	0.1	39.0	4e-08	25000000.0	975000000.0	2.44e+16	0.32	3.154	950000000.0	0.0
0.2	0.0	0.15	16.78	2e-08	56250000.0	943750000.0	5.31e+16	0.488	3.358	887500000.0	0.0
0.2	0.0	0.2	9.0	1e-08	100000000.0	900000000.0	9.00e+16	0.667	3.667	800000000.0	0.0
0.2	0.0	0.25	5.4	1e-08	156250000.0	843750000.0	1.32e+17	0.861	4.111	687500000.0	0.0
0.2	0.0	0.3	3.44	0.0	225000000.0	775000000.0	1.74e+17	1.078	4.742	550000000.0	0.0
0.2	0.0	0.35	2.26	0.0	306250000.0	693750000.0	2.12e+17	1.329	5.649	387500000.0	0.0
0.2	0.0	0.4	1.5	0.0	400000000.0	600000000.0	2.40e+17	1.633	7.0	200000000.0	0.0
0.2	0.05	0.1	49.0	4e-08	25000000.0	1225000000.0	3.06e+16	0.286	3.122	1200000000.0	250000000.0
0.2	0.05	0.15	21.22	2e-08	56250000.0	1193750000.0	6.71e+16	0.434	3.283	1137500000.0	250000000.0

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0.2	0.05	0.2	11.5	1e-08	100000000.0	1150000000.0	1.15e+17	0.59	3.522	1050000000.0	250000000.0
0.2	0.05	0.25	7.0	1e-08	156250000.0	1093750000.0	1.71e+17	0.756	3.857	937500000.0	250000000.0
0.2	0.05	0.3	4.56	0.0	225000000.0	1025000000.0	2.31e+17	0.937	4.317	800000000.0	250000000.0
0.2	0.05	0.35	3.08	0.0	306250000.0	943750000.0	2.89e+17	1.139	4.947	637500000.0	250000000.0
0.2	0.05	0.4	2.12	0.0	400000000.0	850000000.0	3.40e+17	1.372	5.824	450000000.0	250000000.0
0.2	0.1	0.1	59.0	4e-08	25000000.0	1475000000.0	3.69e+16	0.26	3.102	1450000000.0	500000000.0
0.2	0.1	0.15	25.67	2e-08	56250000.0	1443750000.0	8.12e+16	0.395	3.234	1387500000.0	500000000.0
0.2	0.1	0.2	14.0	1e-08	100000000.0	1400000000.0	1.40e+17	0.535	3.429	1300000000.0	500000000.0
0.2	0.1	0.25	8.6	1e-08	156250000.0	1343750000.0	2.10e+17	0.682	3.698	1187500000.0	500000000.0
0.2	0.1	0.3	5.67	0.0	225000000.0	1275000000.0	2.87e+17	0.84	4.059	1050000000.0	500000000.0
0.2	0.1	0.35	3.898	0.0	306250000.0	1193750000.0	3.66e+17	1.013	4.539	887500000.0	500000000.0
0.2	0.1	0.4	2.75	0.0	400000000.0	1100000000.0	4.40e+17	1.206	5.182	700000000.0	500000000.0
0.2	0.2	0.1	79.0	4e-08	25000000.0	1975000000.0	4.94e+16	0.225	3.076	1950000000.0	1000000000.0
0.2	0.2	0.15	34.56	2e-08	56250000.0	1943750000.0	1.09e+17	0.34	3.174	1887500000.0	1000000000.0
0.2	0.2	0.2	19.0	1e-08	100000000.0	1900000000.0	1.90e+17	0.459	3.316	1800000000.0	1000000000.0
0.2	0.2	0.25	11.8	1e-08	156250000.0	1843750000.0	2.88e+17	0.582	3.508	1687500000.0	1000000000.0
0.2	0.2	0.3	7.89	0.0	225000000.0	1775000000.0	3.99e+17	0.712	3.761	1550000000.0	1000000000.0
0.2	0.2	0.35	5.53	0.0	306250000.0	1693750000.0	5.19e+17	0.85	4.085	1387500000.0	1000000000.0
0.2	0.2	0.4	4.0	0.0	400000000.0	1600000000.0	6.4e+17	1.0	4.5	1200000000.0	1000000000.0

NB: r = intrinsic growth rate, m = net migration rate, f is strength of multiplicative noise, K is carrying capacity, k = shape parameter, **β** = Rate parameter, **θ** = Scale parameter.

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Table 3: The parameters and properties of the NKIKE distribution (r = 0.3) and RMSE (at m= 0)

r	m	f	K	B	θ	Mean	Variance	Skewness	Kurtosis	Mode	RMSE (m=0)
0.3	-0.2	0.1	19.0	6e-08	16666666.6667	316666667.0	5.28e+15	0.459	3.316	300000000.0	666666667.0
0.3	-0.2	0.15	7.89	3e-08	37500000.0	295833333.0	1.11e+16	0.712	3.761	258333333.0	666666667.0
0.3	-0.2	0.2	4.0	1e-08	66666666.6667	266666667.0	1.78e+16	1.0	4.5	200000000.0	666666667.0
0.3	-0.2	0.25	2.2	1e-08	104166666.6667	229166667.0	2.39e+16	1.348	5.727	125000000.0	666666667.0
0.3	-0.2	0.3	1.22	1e-08	150000000.0	183333333.0	2.75e+16	1.809	7.909	33333333.0	666666667.0
0.3	-0.2	0.35	0.63	0.0	204166666.6667	129166667.0	2.64e+16	2.514	12.484	0.0	666666667.0
0.3	-0.2	0.4	0.25	0.0	266666666.6667	66666667.0	1.78e+16	4.0	27.0	0.0	666666667.0
0.3	-0.1	0.1	39.0	6e-08	16666666.6667	650000000.0	1.08e+16	0.32	3.154	633333333.0	333333333.0
0.3	-0.1	0.15	16.78	3e-08	37500000.0	629166667.0	2.36e+16	0.488	3.358	591666667.0	333333333.0
0.3	-0.1	0.2	9.0	1e-08	66666666.6667	600000000.0	4.00e+16	0.667	3.667	533333333.0	333333333.0
0.3	-0.1	0.25	5.4	1e-08	104166666.6667	562500000.0	5.859375e+16	0.861	4.111	458333333.0	333333333.0
0.3	-0.1	0.3	3.44	1e-08	150000000.0	516666667.0	7.75e+16	1.078	4.742	366666667.0	333333333.0
0.3	-0.1	0.35	2.26	0.0	204166666.6667	462500000.0	9.44e+16	1.329	5.649	258333333.0	333333333.0
0.3	-0.1	0.4	1.5	0.0	266666666.6667	400000000.0	1.07e+17	1.633	7.0	133333333.0	333333333.0
0.3	-0.05	0.1	49.0	6e-08	16666666.6667	816666667.0	1.36e+16	0.286	3.122	800000000.0	166666667.0
0.3	-0.05	0.15	21.22	3e-08	37500000.0	795833333.0	2.98e+16	0.434	3.283	758333333.0	166666667.0
0.3	-0.05	0.2	11.5	1e-08	66666666.6667	766666667.0	5.11e+16	0.59	3.522	700000000.0	166666667.0
0.3	-0.05	0.25	7.0	1e-08	104166666.6667	729166667.0	7.59e+16	0.756	3.857	625000000.0	166666667.0

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0.3	-0.05	0.3	4.56	1e-08	150000000.0	683333333.0	1.02e+17	0.937	4.317	533333333.0	166666667.0
0.3	-0.05	0.35	3.08	0.0	204166666.6667	629166667.0	1.28e+17	1.139	4.947	425000000.0	166666667.0
0.3	-0.05	0.4	2.125	0.0	266666666.6667	566666667.0	1.51e+17	1.372	5.824	300000000.0	166666667.0
0.3	0.0	0.1	59.0	6e-08	16666666.6667	983333333.0	1.64e+16	0.26	3.102	966666667.0	0.0
0.3	0.0	0.15	25.67	3e-08	37500000.0	962500000.0	3.61e+16	0.395	3.234	925000000.0	0.0
0.3	0.0	0.2	14.0	1e-08	66666666.6667	933333333.0	6.22e+16	0.535	3.429	866666667.0	0.0
0.3	0.0	0.25	8.6	1e-08	104166666.6667	895833333.0	9.33e+16	0.682	3.698	791666667.0	0.0
0.3	0.0	0.3	5.67	1e-08	150000000.0	850000000.0	1.27e+17	0.84	4.059	700000000.0	0.0
0.3	0.0	0.35	3.898	0.0	204166666.6667	795833333.0	1.62e+17	1.013	4.539	591666667.0	0.0
0.3	0.0	0.4	2.75	0.0	266666666.6667	733333333.0	1.96e+17	1.206	5.182	466666667.0	0.0
0.3	0.05	0.1	69.0	6e-08	16666666.6667	1150000000.0	1.92e+16	0.241	3.087	1133333333.0	166666667.0
0.3	0.05	0.15	30.11	3e-08	37500000.0	1129166667.0	4.23e+16	0.364	3.199	1091666667.0	166666667.0
0.3	0.05	0.2	16.5	1e-08	66666666.6667	1100000000.0	7.33e+16	0.492	3.364	1033333333.0	166666667.0
0.3	0.05	0.25	10.2	1e-08	104166666.6667	1062500000.0	1.11e+17	0.626	3.588	958333333.0	166666667.0
0.3	0.05	0.3	6.78	1e-08	150000000.0	1016666667.0	1.52e+17	0.768	3.885	866666667.0	166666667.0
0.3	0.05	0.35	4.71	0.0	204166666.6667	962500000.0	1.96e+17	0.921	4.273	758333333.0	166666667.0
0.3	0.05	0.4	3.375	0.0	266666666.6667	900000000.0	2.40e+17	1.089	4.778	633333333.0	166666667.0
0.3	0.1	0.1	79.0	6e-08	16666666.6667	1316666667.0	2.19e+16	0.225	3.076	1300000000.0	333333333.0
0.3	0.1	0.15	34.56	3e-08	37500000.0	1295833333.0	4.86e+16	0.34	3.174	1258333333.0	333333333.0
0.3	0.1	0.2	19.0	1e-08	66666666.6667	1266666667.0	8.44e+16	0.459	3.316	1200000000.0	333333333.0
0.3	0.1	0.25	11.8	1e-08	104166666.6667	1229166667.0	1.28e+17	0.582	3.508	1125000000.0	333333333.0

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0.3	0.1	0.3	7.89	1e-08	150000000.0	1183333333.0	1.77e+17	0.712	3.761	1033333333.0	333333333.0
0.3	0.1	0.35	5.53	0.0	204166666.6667	1129166667.0	2.31e+17	0.85	4.085	925000000.0	333333333.0
0.3	0.1	0.4	4.0	0.0	266666666.6667	1066666667.0	2.84e+17	1.0	4.5	800000000.0	333333333.0
0.3	0.2	0.1	99.0	6e-08	16666666.6667	1650000000.0	2.75e+16	0.201	3.061	1633333333.0	666666667.0
0.3	0.2	0.15	43.44	3e-08	37500000.0	1629166667.0	6.11e+16	0.303	3.138	1591666667.0	666666667.0
0.3	0.2	0.2	24.0	1e-08	66666666.6667	1600000000.0	1.07e+17	0.408	3.25	1533333333.0	666666667.0
0.3	0.2	0.25	15.0	1e-08	104166666.6667	1562500000.0	1.63e+17	0.516	3.4	1458333333.0	666666667.0
0.3	0.2	0.3	10.11	1e-08	150000000.0	1516666667.0	2.27e+17	0.629	3.593	1366666667.0	666666667.0
0.3	0.2	0.35	7.16	0.0	204166666.6667	1462500000.0	2.99e+17	0.747	3.838	1258333333.0	666666667.0
0.3	0.2	0.4	5.25	0.0	266666666.6667	1400000000.0	3.73e+17	0.873	4.143	1133333333.0	666666667.0

NB: r = intrinsic growth rate, m = net migration rate, f is strength of multiplicative noise, K is carrying capacity, k = shape parameter, **β** = Rate parameter, **θ** = Scale parameter.

Table 4: The parameters and properties of the NKIKE distribution (r = 0.4) and RMSE (at m= 0)

r	**m**	**f**	**k**	**B**	**θ**	**Mean**	**Variance**	**Skewness**	**Kurtosis**	**Mode**	**RMSE (m=0)**
0.4	-0.2	0.1	39.0	8e-08	12500000.0	487500000.0	6.09e+15	0.32	3.154	475000000.0	500000000.0
0.4	-0.2	0.15	16.78	4e-08	28125000.0	471875000.0	1.33e+16	0.488	3.358	443750000.0	500000000.0
0.4	-0.2	0.2	9.0	2e-08	50000000.0	450000000.0	2.25e+16	0.667	3.667	400000000.0	500000000.0
0.4	-0.2	0.25	5.4	1e-08	78125000.0	421875000.0	3.30e+16	0.861	4.111	343750000.0	500000000.0
0.4	-0.2	0.3	3.44	1e-08	112500000.0	387500000.0	4.36e+16	1.078	4.742	275000000.0	500000000.0
0.4	-0.2	0.35	2.26	1e-08	153125000.0	346875000.0	5.31e+16	1.329	5.649	193750000.0	500000000.0
0.4	-0.2	0.4	1.5	0.0	200000000.0	300000000.0	6.00e+16	1.633	7.0	100000000.0	500000000.0
0.4	-0.1	0.1	59.0	8e-08	12500000.0	737500000.0	9.22e+15	0.26	3.102	725000000.0	250000000.0

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0.4	-0.1	0.15	25.67	4e-08	28125000.0	721875000.0	2.03e+16	0.395	3.234	693750000.0	250000000.0
0.4	-0.1	0.2	14.00	2e-08	50000000.0	700000000.0	3.50e+16	0.535	3.429	650000000.0	250000000.0
0.4	-0.1	0.25	8.60	1e-08	78125000.0	671875000.0	5.25e+16	0.682	3.698	593750000.0	250000000.0
0.4	-0.1	0.3	5.67	1e-08	112500000.0	637500000.0	7.17e+16	0.84	4.059	525000000.0	250000000.0
0.4	-0.1	0.35	3.90	1e-08	153125000.0	596875000.0	9.14e+16	1.013	4.539	443750000.0	250000000.0
0.4	-0.1	0.4	2.75	0.0	200000000.0	550000000.0	1.10e+17	1.206	5.182	350000000.0	250000000.0
0.4	-0.05	0.1	69.0	8e-08	12500000.0	862500000.0	1.08e+16	0.241	3.087	850000000.0	125000000.0
0.4	-0.05	0.15	30.11	4e-08	28125000.0	846875000.0	2.38e+16	0.364	3.199	818750000.0	125000000.0
0.4	-0.05	0.2	16.5	2e-08	50000000.0	825000000.0	4.13e+16	0.492	3.364	775000000.0	125000000.0
0.4	-0.05	0.25	10.2	1e-08	78125000.0	796875000.0	6.23e+16	0.626	3.588	718750000.0	125000000.0
0.4	-0.05	0.3	6.78	1e-08	112500000.0	762500000.0	8.58e+16	0.768	3.885	650000000.0	125000000.0
0.4	-0.05	0.35	4.71	1e-08	153125000.0	721875000.0	1.11e+17	0.921	4.273	568750000.0	125000000.0
0.4	-0.05	0.4	3.38	0.0	200000000.0	675000000.0	1.35e+17	1.089	4.778	475000000.0	125000000.0
0.4	0.0	0.1	79.0	8e-08	12500000.0	987500000.0	1.23e+16	0.225	3.076	975000000.0	0.0
0.4	0.0	0.15	34.56	4e-08	28125000.0	971875000.0	2.73e+16	0.34	3.174	943750000.0	0.0
0.4	0.0	0.2	19.00	2e-08	50000000.0	950000000.0	4.75e+16	0.459	3.316	900000000.0	0.0
0.4	0.0	0.25	11.80	1e-08	78125000.0	921875000.0	7.20e+16	0.582	3.508	843750000.0	0.0
0.4	0.0	0.3	7.89	1e-08	112500000.0	887500000.0	9.98e+16	0.712	3.761	775000000.0	0.0
0.4	0.0	0.35	5.53	1e-08	153125000.0	846875000.0	1.30e+17	0.85	4.085	693750000.0	0.0
0.4	0.0	0.4	4.0	0.0	200000000.0	800000000.0	1.60e+17	1.0	4.5	600000000.0	0.0
0.4	0.05	0.1	89.0	8e-08	12500000.0	1112500000.0	1.39e+16	0.212	3.067	1100000000.0	125000000.0

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0.4	0.05	0.15	39.0	4e-08	28125000.0	1096875000.0	3.08e+16	0.32	3.154	1068750000.0	125000000.0
0.4	0.05	0.2	21.5	2e-08	50000000.0	1075000000.0	5.37e+16	0.431	3.279	1025000000.0	125000000.0
0.4	0.05	0.25	13.4	1e-08	78125000.0	1046875000.0	8.18e+16	0.546	3.448	968750000.0	125000000.0
0.4	0.05	0.3	9.0	1e-08	112500000.0	1012500000.0	1.14e+17	0.667	3.667	900000000.0	125000000.0
0.4	0.05	0.35	6.35	1e-08	153125000.0	971875000.0	1.49e+17	0.794	3.945	818750000.0	125000000.0
0.4	0.05	0.4	4.63	0.0	200000000.0	925000000.0	1.85e+17	0.93	4.297	725000000.0	125000000.0
0.4	0.1	0.1	99.0	8e-08	12500000.0	1237500000.0	1.55e+16	0.201	3.061	1225000000.0	250000000.0
0.4	0.1	0.15	43.44	4e-08	28125000.0	1221875000.0	3.44e+16	0.303	3.138	1193750000.0	250000000.0
0.4	0.1	0.2	24.0	2e-08	50000000.0	1200000000.0	6.00e+16	0.408	3.25	1150000000.0	250000000.0
0.4	0.1	0.25	15.0	1e-08	78125000.0	1171875000.0	9.16e+16	0.516	3.4	1093750000.0	250000000.0
0.4	0.1	0.3	10.11	1e-08	112500000.0	1137500000.0	1.28e+17	0.629	3.593	1025000000.0	250000000.0
0.4	0.1	0.35	7.16	1e-08	153125000.0	1096875000.0	1.68e+17	0.747	3.838	943750000.0	250000000.0
0.4	0.1	0.4	5.25	0.0	200000000.0	1050000000.0	2.1e+17	0.873	4.143	850000000.0	250000000.0
0.4	0.2	0.1	119.0	8e-08	12500000.0	1487500000.0	1.86e+16	0.183	3.05	1475000000.0	500000000.0
0.4	0.2	0.15	52.33	4e-08	28125000.0	1471875000.0	4.14e+16	0.276	3.115	1443750000.0	500000000.0
0.4	0.2	0.2	29.0	2e-08	50000000.0	1450000000.0	7.25e+16	0.371	3.207	1400000000.0	500000000.0
0.4	0.2	0.25	18.2	1e-08	78125000.0	1421875000.0	1.11e+17	0.469	3.33	1343750000.0	500000000.0
0.4	0.2	0.3	12.33	1e-08	112500000.0	1387500000.0	1.56e+17	0.569	3.486	1275000000.0	500000000.0
0.4	0.2	0.35	8.80	1e-08	153125000.0	1346875000.0	2.06e+17	0.674	3.682	1193750000.0	500000000.0
0.4	0.2	0.4	6.50	0.0	200000000.0	1300000000.0	2.60e+17	0.784	3.923	1100000000.0	500000000.0

NB: r = intrinsic growth rate, m = net migration rate, f is strength of multiplicative noise, K is carrying capacity, k = shape parameter, **β** = Rate parameter, **θ** = Scale parameter.

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Table 5: The parameters and properties of the NKIKE distribution (r = 0.5) and RMSE (at m= 0)

r	m	F	k	B	Θ	Mean	Variance	Skewness	Kurtosis	Mode	RMSE (m=0)
0.5	-0.2	0.1	59.0	1e-07	10000000.0	590000000.0	5.90e+15	0.26	3.102	580000000.0	400000000.0
0.5	-0.2	0.15	25.67	4e-08	22500000.0	577500000.0	1.30e+16	0.395	3.234	555000000.0	400000000.0
0.5	-0.2	0.2	14.0	2e-08	40000000.0	560000000.0	2.24e+16	0.535	3.429	520000000.0	400000000.0
0.5	-0.2	0.25	8.6	2e-08	62500000.0	537500000.0	3.36e+16	0.682	3.698	475000000.0	400000000.0
0.5	-0.2	0.3	5.67	1e-08	90000000.0	510000000.0	4.59e+16	0.84	4.059	420000000.0	400000000.0
0.5	-0.2	0.35	3.90	1e-08	122500000.0	477500000.0	5.85e+16	1.013	4.539	355000000.0	400000000.0
0.5	-0.2	0.4	2.75	1e-08	160000000.0	440000000.0	7.04e+16	1.206	5.182	280000000.0	400000000.0
0.5	-0.1	0.1	79.0	1e-07	10000000.0	790000000.0	7.90e+15	0.225	3.076	780000000.0	200000000.0
0.5	-0.1	0.15	34.56	4e-08	22500000.0	777500000.0	1.75e+16	0.34	3.174	755000000.0	200000000.0
0.5	-0.1	0.2	19.0	2e-08	40000000.0	760000000.0	3.04e+16	0.459	3.316	720000000.0	200000000.0
0.5	-0.1	0.25	11.8	2e-08	62500000.0	737500000.0	4.61e+16	0.582	3.508	675000000.0	200000000.0
0.5	-0.1	0.3	7.89	1e-08	90000000.0	710000000.0	6.39e+16	0.712	3.761	620000000.0	200000000.0
0.5	-0.1	0.35	5.53	1e-08	122500000.0	677500000.0	8.30e+16	0.85	4.085	555000000.0	200000000.0
0.5	-0.1	0.4	4.0	1e-08	160000000.0	640000000.0	1.02e+17	1.0	4.5	480000000.0	200000000.0
0.5	-0.05	0.1	89.0	1e-07	10000000.0	890000000.0	8.90e+15	0.212	3.067	880000000.0	100000000.0
0.5	-0.05	0.15	39.0	4e-08	22500000.0	877500000.0	1.97e+16	0.32	3.154	855000000.0	100000000.0
0.5	-0.05	0.2	21.5	2e-08	40000000.0	860000000.0	3.44e+16	0.431	3.279	820000000.0	100000000.0
0.5	-0.05	0.25	13.4	2e-08	62500000.0	837500000.0	5.23e+16	0.546	3.448	775000000.0	100000000.0
0.5	-0.05	0.3	9.0	1e-08	90000000.0	810000000.0	7.29e+16	0.667	3.667	720000000.0	100000000.0

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0.5	-0.05	0.35	6.35	1e-08	122500000.0	777500000.0	9.52e+16	0.794	3.945	655000000.0	100000000.0
0.5	-0.05	0.4	4.63	1e-08	160000000.0	740000000.0	1.18e+17	0.93	4.297	580000000.0	100000000.0
0.5	0.0	0.1	99.0	1e-07	10000000.0	990000000.0	9.90e+15	0.201	3.061	980000000.0	0.0
0.5	0.0	0.15	43.44	4e-08	22500000.0	977500000.0	2.20e+16	0.303	3.138	955000000.0	0.0
0.5	0.0	0.2	24.0	2e-08	40000000.0	960000000.0	3.84e+16	0.408	3.25	920000000.0	0.0
0.5	0.0	0.25	15.0	2e-08	62500000.0	937500000.0	5.86e+16	0.516	3.4	875000000.0	0.0
0.5	0.0	0.3	10.11	1e-08	90000000.0	910000000.0	8.19e+16	0.629	3.593	820000000.0	0.0
0.5	0.0	0.35	7.16	1e-08	122500000.0	877500000.0	1.07e+17	0.747	3.838	755000000.0	0.0
0.5	0.0	0.4	5.25	1e-08	160000000.0	840000000.0	1.34e+17	0.873	4.143	680000000.0	0.0
0.5	0.05	0.1	109.0	1e-07	10000000.0	1090000000.0	1.09e+16	0.192	3.055	1080000000.0	100000000.0
0.5	0.05	0.15	47.89	4e-08	22500000.0	1077500000.0	2.42e+16	0.289	3.125	1055000000.0	100000000.0
0.5	0.05	0.2	26.5	2e-08	40000000.0	1060000000.0	4.24e+16	0.389	3.226	1020000000.0	100000000.0
0.5	0.05	0.25	16.6	2e-08	62500000.0	1037500000.0	6.48e+16	0.491	3.361	975000000.0	100000000.0
0.5	0.05	0.3	11.22	1e-08	90000000.0	1010000000.0	9.09e+16	0.597	3.535	920000000.0	100000000.0
0.5	0.05	0.35	7.98	1e-08	122500000.0	977500000.0	1.20e+17	0.708	3.752	855000000.0	100000000.0
0.5	0.05	0.4	5.88	1e-08	160000000.0	940000000.0	1.50e+17	0.825	4.021	780000000.0	100000000.0
0.5	0.1	0.1	119.0	1e-07	10000000.0	1190000000.0	1.19e+16	0.183	3.05	1180000000.0	200000000.0
0.5	0.1	0.15	52.33	4e-08	22500000.0	1177500000.0	2.65e+16	0.276	3.115	1155000000.0	200000000.0
0.5	0.1	0.2	29.0	2e-08	40000000.0	1160000000.0	4.64e+16	0.371	3.207	1120000000.0	200000000.0
0.5	0.1	0.25	18.2	2e-08	62500000.0	1137500000.0	7.11e+16	0.469	3.33	1075000000.0	200000000.0
0.5	0.1	0.3	12.33	1e-08	90000000.0	1110000000.0	9.99e+16	0.569	3.486	1020000000.0	200000000.0

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0.5	0.1	0.35	8.80	1e-08	122500000.0	1077500000.0	1.32e+17	0.674	3.682	955000000.0	200000000.0
0.5	0.1	0.4	6.5	1e-08	160000000.0	1040000000.0	1.66e+17	0.784	3.923	880000000.0	200000000.0
0.5	0.2	0.1	139.0	1e-07	10000000.0	1390000000.0	1.39e+16	0.17	3.043	1380000000.0	400000000.0
0.5	0.2	0.15	61.22	4e-08	22500000.0	1377500000.0	3.10e+16	0.256	3.098	1355000000.0	400000000.0
0.5	0.2	0.2	34.0	2e-08	40000000.0	1360000000.0	5.44e+16	0.343	3.176	1320000000.0	400000000.0
0.5	0.2	0.25	21.4	2e-08	62500000.0	1337500000.0	8.36e+16	0.432	3.28	1275000000.0	400000000.0
0.5	0.2	0.3	14.56	1e-08	90000000.0	1310000000.0	1.18e+17	0.524	3.412	1220000000.0	400000000.0
0.5	0.2	0.35	10.43	1e-08	122500000.0	1277500000.0	1.56e+17	0.619	3.575	1155000000.0	400000000.0
0.5	0.2	0.4	7.75	1e-08	160000000.0	1240000000.0	1.98e+17	0.718	3.774	1080000000.0	400000000.0

NB: r = intrinsic growth rate, m = net migration rate, f is strength of multiplicative noise, K is carrying capacity, k = shape parameter, **β** = Rate parameter, **θ** = Scale parameter.

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Discussion

The results of the simulated model (Tables 1 – 5) show that reducing the drift term by either lowering r or net m or both will slow down the deterministic growth, while controlling diffusion with a moderate increase in f can flatten the pdf and decrease the probability concentration of high population size P.

Hence, Tables 1 to 5 reveal that higher values of r increase the shape parameter k, leading to a more peaked and stable distribution with lower variance and skewness, while lower r values tend to produce flatter, left-skewed distributions unless supported by migration. This implies that a high fertility rate will imply more predictable population growth if migration volatility is low, which aligns with Tuljapurkar (1990), who emphasised the role of demographic stochasticity in modulating population inertia in low-fertility contexts (Aitken, 2024; Feng, Zhou, Qiu, & Kang, 2022; Compagnoni et al., 2016; Tuljapurkar, 1990).

Similarly, Tables 1 – 5 show that positive m increases the total growth rate (r + m), increasing both the shape parameter k and the expected value, while negative m reduces the shape k, leading to higher skewness and potentially bimodal or unstable behaviour at low r. The RMSE comparison against the m = 0 baseline shows that moderate positive m tends to reduce RMSE when r is low and negative m increases deviation significantly. This implies that migration inflows can buffer low fertility while outflows can destabilise population equilibrium, especially under low r and high f, which aligns with other studies that showed that migration modifies the expected age structure and variance in long-term forecasts (Sevcikova, Raymer, & Raftery, 2024; Fuchs, Sohnlein, & Vanella, 2021; Wisniowski, 2021; Lee & Miller, 2001).

In addition, Tables 1 – 5 show that higher stochastic fluctuation f increases the denominator in both the shape k and scale θ, leading to higher shape k and larger scale θ, leading to broader and flatter PDFs with greater uncertainty and increased RMSE. Also, under high f, even high values of r or m cannot fully stabilise the population distribution. This implies that environmental volatility can overwhelm demographic efforts unless mitigated by policies (e.g., health access and social stability), and this aligns with studies which noted that demographic variability becomes dominant in small or unstable populations with environmental noise (Gaynor et al., 2025; Jonsson & Wennergren, 2019; Greenman & Benton, 2003; Lande, 1993). Hence, balanced fertility r and moderate migration m result in optimal distributions with low RSME and moderate variance; high stochastic environments require more conservative planning with buffers (like health and food security), and outmigration policies (rural-urban drift reduction) should account for destabilising effects on local population growth

The simulated results of the migration-modified stochastic logistic growth models and their probability distribution (the NKIKE distribution) do show that reducing the intrinsic growth rate by implementing family planning (through expanded access to contraceptives and health campaigns), increasing education (especially among females) and raising legal marriage age will help slow down population growth. Also, enforcing migration controls (through border policies and incentivising return migration) or managing urbanisation (through regional development of origin areas) and reducing positive inflow from other areas will lower the net migration and help control population growth. In addition, population control can be implemented by limiting the carrying capacity (K) through restricting access to infrastructure (limiting housing and agricultural expansion) and reducing environmental or economic support that sustains large family sizes and large populations. Lastly, increasing environmental noise by instituting regulations (economic reforms and uncertain policy environments) could introduce variability (e.g., in employment and income) in the population, thereby increasing unpredictability in population outcomes, causing fluctuations and lower expected sizes (DuPuy, Galaitsi, & Linkov, 2025; Kortessis et al., 2024; Senderowicz & Valley, 2023; Hoffmann & Muttarak, 2021; Liu & Raftery, 2020; Bao et al., 2020;

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Clemens & Postel, 2018).

In countries with limited data infrastructure, weak governance and high demographic pressure, deterministic forecasts are often unreliable. The approach proposed in this study offers a probabilistic alternative, one that incorporates variability, uncertainty and nonlinearity. This approach is relevant for Nigeria, with one of the fastest-growing populations globally, and where internal migration and urbanisation have intensified (NPC, 2023); others like Ethiopia facing demographic transition and rural-urban migration under climate stress (World Bank, 2022) and those where environmental displacement reshapes population trends. The MM-SLGM, by offering an adaptable framework, aligns with international efforts such as the UN's Sustainable Development Goals (SDG 3 and SDG 11) on health and sustainable cities and also supports data-driven planning in low- and middle-income countries.

Conclusion

The study confirmed that population stability is a function of internal fertility dynamics (r), external mobility (m) and environmental risk (f) and that long-term policies must target these three levers simultaneously to maintain demographic equilibrium and avoid either explosive growth or population collapse. Hence, there is a need to adopt a coordinated strategy that simultaneously moderates fertility, balances migration and stabilises environmental noise to avoid population growth and underperformance. Calibrating population growth via balanced fertility policies (control r in the range 0.2 – 0.3), regulate migration to maintain demographic balance (manage m in the range 0 – 0.1), mitigating environmental and socioeconomic volatility (stabilise $f < 0.25$), embedding stochastic modelling in national planning and monitoring and evaluation the models using RMSE metrics will ensure population management and planning particularly in a developing context like Nigeria.

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