Published by

British Journal of Contemporary Research

BEXFORD PUBLISHING

Bayesian Inference in Machine Interference Model

Original Research Article | Volume 1 | Issue 3 | 2026 | Article Number: 058

Accepted: 05 August 2026 | Published: 10 August 2026 | ISSN: 2979-8582 (Online)

Crossref : **https://doi.org/10.67693/BJCR-BJV4KTA4**

Bidyut Das[1], Amit Choudhury[2]

[1]Department of Statistics, Nagaon University, Nagaon, India,
[2]Department of Statistics, Gauhati University, Guwahati, India

BD 0000-0001-6686-2752

Correspondence: Bidyut Das, bidyutindrani@gmail.com

Abstract

This paper investigates Bayesian estimation of the performance measures of a machine inference queuing model under a binomial distribution-based framework, assuming the system operates in steady state. The primary parameters of interest are the traffic intensity ρ and the expected number of customers in the system Ls. Three prior distributions are considered: a Beta prior of the second kind (informative, natural conjugate), a Gamma prior (informative, non-conjugate), and Jeffreys' prior (non-informative). Bayesian estimators for ρ and Ls are derived under two loss functions — the Squared Error Loss Function (SELF) and the Precautionary Loss Function (PLF) — together with their corresponding posterior risks. Equal-tailed credible intervals for ρ are obtained from all four posterior distributions. Predictive distributions for future observations are derived under each prior, enabling posterior model comparison via Bayes factors following Jeffreys' (1998) decision rule. Relative efficiencies are assessed using mean squared error (MSE) via Monte Carlo simulation across different sample sizes and traffic intensities. Results indicate that all estimators converge to the true value as n increases, that the informative Beta prior consistently yields lower posterior risk when hyperparameters are well-specified, and that PLF estimators are uniformly larger than their SELF counterparts, providing a conservative guard against underestimation.

Keywords: Bayesian Inference, Traffic Intensity, Beta Prior, Jeffreys Prior, SELF, PLF, Posterior Risk, Monte Carlo Simulation

INTRODUCTION

Waiting-line phenomena pervade everyday life: customers queuing at a supermarket checkout, aircraft circling before landing, or machines waiting for a repair crew all share the same underlying structure. Queueing theory provides the mathematical machinery to describe such systems, and a central practical concern is the estimation of their performance measures—quantities such as the traffic intensity, the mean queue length and the expected number of customers in the system—from observed data. Because these measures are non-linear functions of the arrival and service rates, their estimation is delicate, and both classical and Bayesian frameworks have been employed to address it.

The classical treatment of the M/M/1 system dates back to Clarke (1957), who derived estimates of the arrival and service rates λ and μ. Muddapur (1972) placed a prior distribution over Clarke's model and obtained the first Bayes estimators, and Reynolds (1973) extended the Bayesian viewpoint to birth–death processes. Subsequent work broadened the scope considerably: McGrath, Gross and Singpurwalla (1987) illustrated the Bayesian approach on M/M/1 and M/M/1/K queues, while Armero and Bayarri (1994, 1997) studied steady-state and predictive inference for M/M/1 and M/M/∞ systems. Bayesian inference has since been developed for M/G/1 queues (Insua, Wiper and Ruggeri, 1998), discrete-time Geo/G/1 systems (Conti, 1999), bulk-arrival queues (Armero and Conesa, 2000) and GI/M/c designs (Ausin, Lillo and Wiper, 2007). Choudhury and Borthakur (2008) derived Bayesian inferences for the performance measures of the M/M/1 queue using conjugate and truncated-uniform priors, and more recently Basak and Choudhury (2021) obtained inferences for the M/M/1 and M/M/s systems based on queue length.

Despite this rich literature, comparatively little attention has been paid to the machine-interference (finite-source) model under a range of loss functions and prior specifications. The present paper fills this gap. Assuming the system is in steady state, we estimate the traffic intensity ρ and the expected number of customers L□ under three priors—a beta prior of the second kind, a gamma prior, and Jeffreys' prior and under two loss functions, the symmetric squared-error loss and the asymmetric precautionary loss. Equal-tailed credible intervals, posterior predictive distributions and Bayes factors are also developed, and the estimators are compared with their classical counterparts through a comprehensive simulation study. The remainder of the chapter sets out the model and the four priors, derives the Bayes estimators together with their posterior risks under each loss, and reports the numerical findings before concluding.

1. Model Description

We have shown in chapter 4, that in the M/M/k//k model

$$p_x = \binom{k}{x}\left(\frac{\rho}{1+\rho}\right)^x\left(\frac{1}{1+\rho}\right)^{k-x}, \quad x = 0,1,2,\ldots,k;\ \rho > 0$$

which follows binomial distribution with parameters k and $\frac{\rho}{1+\rho}$.

Our interest is in the parameter ρ.

The $MLE\ of$ ρ $is\ \ given\ by$ $\dfrac{\sum_{i=1}^{n} x_i}{nk-\sum_{i=1}^{n} x_i}$

2. Preliminaries

In this paper, assuming that the queuing system is in the steady state and under the Bayesian setup here, we estimate the parameter ρ and the expected number of customers L_s in the system. The main objective of the researcher to use Bayesian statistical inference is to derive decision rules under a specified loss function and a prior distribution over the parameter space. Here, we have taken one natural conjugate informative prior (beta prior second kind), one non-conjugate informative prior (Gamma prior) and one non-informative priors (Jeffrey's prior). Here, our parameters of interest are estimated under two commonly used loss functions (Zaka and Akhter 2014), as follows.

2.1 Squared error loss function

The square error loss function (SELF) is given by

$$L_1(\hat{\varphi}, \varphi) = (\hat{\varphi} - \varphi)^2$$

where $\hat{\varphi}$ is an estimate of φ. This loss function is symmetrical in nature and assigns equal losses to both overestimation and underestimation.

For a random sample of size n, $x = \{x_1, x_2, ..., x_n\}$, the Bayesian estimator of φ under the square error loss function, denoted by $\hat{\varphi}_{SELF}$, can be obtained as

$$\hat{\varphi}_{SELF} = E(\varphi|x)$$

and the posterior risk associated with $\hat{\varphi}_{SELF}$ is

$$R_1(\hat{\varphi}_{SELF}, \varphi|x) = V(\varphi|x) = E(\varphi^2|x) - \{E(\varphi|x)\}^2$$

2.2 Precautionary loss function

We very often face undesirable situations as a result of overestimation and underestimation of the symmetrical loss function. Thus, it is inappropriate if we use the symmetric loss function. To overcome this situation, Norstrom (1996) proposed an asymmetric loss function known as precautionary loss function (PLF) and is given by

$$L_2(\hat{\varphi}, \varphi) = \frac{(\hat{\varphi}-\varphi)^2}{\hat{\varphi}}$$

For a random sample of size n, $x = \{x_1, x_2, ..., x_n\}$, the Bayesian estimator of φ under the precautionary loss function, denoted by $\hat{\varphi}_{PLF}$, can be obtained as

$$\hat{\varphi}_{PLF} = [E(\varphi^2|x)]^{\frac{1}{2}}$$

and the posterior risk associated with $\hat{\varphi}_{PLF}$ is

$$R_2(\hat{\varphi}_{PLF}, \varphi|x) = 2\left\{[E(\varphi^2|x)]^{\frac{1}{2}} - E(\varphi|x)\right\} = 2(\hat{\varphi}_{PLF} - \hat{\varphi}_{SELF})$$

3. Beta Prior

Generally, priors are chosen so that the range of the studied parameter matches that of the distribution. In our model ρ is greater than zero, i.e., $\rho > 0$ and in the Bayesian framework, the Beta distribution of the second kind could be suitable prior for ρ.

The probability density of the Beta prior is given by

$$q_1(\rho|c,d) = \frac{1}{\beta(c,d)}\frac{\rho^{c-1}}{(1+\rho)^{c+d}},\ \ \rho > 0 \tag{3}$$

Here the Beta function, $\beta(c, d)$, is the normalization constant that ensures the total probability is equal to 1; which is defined as $\beta(c,d) = \int_0^\infty \frac{\rho^{c-1}}{(1+\rho)^{c+d}} d\rho$

Here c and d are known as hyper-parameters and $c, d > 0$.

For a random sample of size n, $x = \{x_1, x_2, ..., x_n\}$, the posterior distribution of ρ for the prior $q_1(\rho|c,d)$, from equation (3), is calculated as

$$p_1(\rho|x,c,d) = \frac{q_1(\rho|c,d)\times L(\rho,x)}{\int_0^\infty q_1(\rho|c,d)\times L(\rho,x)d\rho}$$

$$= \frac{\frac{1}{\beta(c,d)}\frac{\rho^{c-1}}{(1+\rho)^{c+d}}\times\prod_{i=1}^{k}\binom{k}{x}\left(\frac{\rho}{1+\rho}\right)^{x}\left(\frac{1}{1+\rho}\right)^{k-x}}{\int_0^\infty \frac{1}{\beta(c,d)}\frac{\rho^{c-1}}{(1+\rho)^{c+d}}\times\prod_{i=1}^{k}\binom{k}{x}\left(\frac{\rho}{1+\rho}\right)^{x}\left(\frac{1}{1+\rho}\right)^{k-x}d\rho}$$

$$= \frac{1}{\beta(c+y,d+nk-y)}\frac{\rho^{c+y-1}}{(1+\rho)^{c+d+nk}}$$

where $y = \sum_{i=1}^{n} x_i$

3.1 Estimation of ρ $and\ L_s$ under the SELF:

Bayesian estimate of ρ denoted by $\hat{\rho}^{BP}_{SELF}$ for a random sample of size n, $x = \{x_1, x_2, ..., x_n\}$, is given by

$$\hat{\rho}^{BP}_{SELF} = E(\rho|x,c,d)$$

$$= \int_0^\infty \rho\, p_1(\rho|x,c,d)d\rho$$

$$= \frac{\beta(c+y+1,d+nk-y-1)}{\beta(c+y,d+nk-y)}$$

$$= \frac{(c+y)}{(d+nk-y-1)}$$

with the associated posterior risk as follows

$$R_1\left(\hat{\rho}^{BP}_{SELF}, \rho|x,c,d\right) = V(\rho|x,c,d)$$

$$= E\left(\rho^2|x,c,d\right) - \{E(\rho|x,c,d)\}^2$$

$$= \frac{(c+y)}{(d+nk-y-1)^2}\frac{(c+y+nk-1)}{(d+nk-y-2)}$$

Bayesian estimate of expected number of customers in the system L_s, denoted by $\hat{L}_{s_{SELF}}^{BP}$ is given by

$$\hat{L}_{s_{SELF}}^{BP} = E\left(L_s|x, c, d\right) = E\left(\frac{k\rho}{1+\rho}|x, c, d\right)$$

$$= \int_0^\infty \frac{k\rho}{1+\rho}\, p_1(\rho|x, c, d)d\rho$$

$$= \frac{k}{\beta(c+y,d+nk-y)}\int_0^\infty \frac{\rho^{c+y+1-1}}{(1+\rho)^{c+d+nk+1}}d\rho$$

$$= \frac{k(c+y)}{(c+d+nk)}$$

3.2 Estimation of ρ and L_s under the PLF:

Bayesian estimate of ρ denoted by $\hat{\rho}_{PLF}^{BP}$ for a random sample of size n, $x = \left\{x_1, x_2, ..., x_n\right\}$, is given by

$$\hat{\rho}_{PLF}^{BP} = \left[E\left(\rho^2|x, c, d\right)\right]^{\frac{1}{2}}$$

$$= \left[\int_0^\infty \rho^2\, p_1(\rho|x, c, d)d\rho\right]^{\frac{1}{2}}$$

$$= \left[\frac{\beta(c+y+2,\ d+nk-y-2)}{\beta(c+y,\ d+nk-y)}\right]^{\frac{1}{2}}$$

$$= \left[\frac{(c+y)}{(d+nk-y-1)}\frac{(c+y+1)}{(d+nk-y-2)}\right]^{\frac{1}{2}}$$

with the associated posterior risk as follows

$$R_2\left(\hat{\rho}_{PLF}^{BP}, \rho|x, c, d\right) = 2\left[\hat{\rho}_{PLF}^{BP} - \hat{\rho}_{SELF}^{BP}\right]$$

Bayesian estimate of expected number of customers in the system L_s, denoted by $\hat{L}_{s_{PLF}}^{BP}$ is given by

$$\hat{L}_{s_{PLF}}^{BP} = \left[E\left(L_s^{\ 2}|x, c, d\right)\right]^{\frac{1}{2}}$$

$$= \left[E\left(\left(\frac{k\rho}{1+\rho}\right)^2|x, c, d\right)\right]^{\frac{1}{2}}$$

$$= \left[\int_0^\infty \left(\frac{k\rho}{1+\rho}\right)^2 p_1(\rho|x, c, d)d\rho\right]^{\frac{1}{2}}$$

$$= \left[k^2 \frac{\beta(c+y+2,\ d+nk-y)}{\beta(c+y,\ d+nk-y)}\right]^{\frac{1}{2}}$$

$$= k\left[\frac{(c+y)(c+y+1)}{(c+d+nk)(c+d+nk+1)}\right]^{\frac{1}{2}}$$

4. Gamma Prior:

Another prior distribution for ρ is the gamma distribution whose probability density function is given by

$$q_2(\rho|a, b) = \frac{b^a}{\Gamma(a)}\rho^{a-1}e^{-b\rho},\ \ \rho > 0 \qquad (4)$$

For a random sample of size n, $x = \{x_1, x_2, ..., x_n\}$, the posterior distribution of ρ for the prior $q_2(\rho|a, b)$, from equation (4), is calculated as

$$p_2(\rho|x, a, b) = \frac{q_2(\rho|a,b)\times L(\rho,x)}{\int_0^\infty q_2(\rho|a,b)\times L(\rho,x)d\rho}$$

$$= \frac{\frac{b^a}{\Gamma(a)}\rho^{a-1}e^{-b\rho}\times\prod_{i=1}^{k}\left(\frac{k}{x}\right)\left(\frac{\rho}{1+\rho}\right)^x\left(\frac{1}{1+\rho}\right)^{k-x}}{\int_0^\infty\frac{b^a}{\Gamma(a)}\rho^{a-1}e^{-b\rho}\times\prod_{i=1}^{k}\left(\frac{k}{x}\right)\left(\frac{\rho}{1+\rho}\right)^x\left(\frac{1}{1+\rho}\right)^{k-x}d\rho}$$

$$= \frac{e^{-b\rho}\rho^{a+y-1}(1+\rho)^{-nk}}{\int_0^\infty e^{-b\rho}\rho^{a+y-1}(1+\rho)^{-nk}d\rho}$$

$$= \frac{e^{-b\rho}\rho^{a+y-1}(1+\rho)^{-nk}}{\Gamma(a+y)\cup\ (a+y,a+y-nk+1,b)}$$

where $y = \sum_{i=1}^{n} x_i$ and $\cup\ (a, b, z) = \frac{1}{\Gamma(a)}\int_0^\infty e^{-zt}t^{a-1}(1+t)^{b-a-1}dt$ is the Kummer's confluent hypergeometric function of second kind which is introduced by Kummer (1837)

4.1 Estimation of ρ $and\ L_s$ under the SELF:

Bayesian estimate of ρ denoted by $\hat{\rho}^{GP}_{SELF}$ for a random sample of size n, $x = \{x_1, x_2, ..., x_n\}$, is given by

$$\hat{\rho}^{GP}_{SELF} = E(\rho|x, a, b)$$

$$= \int_0^\infty \rho\ p_2(\rho|x, a, b)d\rho$$

$$= \int_0^\infty \rho\frac{e^{-b\rho}\rho^{a+y-1}(1+\rho)^{-nk}}{\Gamma(a+y)\cup\ (a+y,a+y-nk+1,b)}d\rho$$

$$= \frac{\Gamma(a+y+1)\cup\ (a+y+1,a+y-nk+2,b)}{\Gamma(a+y)\cup\ (a+y,a+y-nk+1,b)}$$

with the associated posterior risk as follows

$$R_1\left(\hat{\rho}^{GP}_{SELF}, \rho|x, a, b\right) = V(\rho|x, a, b)$$

$$= E\left(\rho^2|x, a, b\right) - \{E(\rho|x, a, b)\}^2$$

$$= \int_0^\infty \rho^2\, p_2(\rho|x, a, b)d\rho - \left(\hat{\rho}^{GP}_{SELF}\right)^2$$

$$= \frac{\Gamma(a+y+2)U\ (a+y+2, a+y-nk+3, b)}{\Gamma(a+y)U\ (a+y, a+y-nk+1, b)} - \left(\hat{\rho}^{GP}_{SELF}\right)^2$$

Bayesian estimate of expected number of customers in the system L_s, denoted by $\hat{L}^{GP}_{s_{SELF}}$ is given by

$$\hat{L}^{GP}_{s_{SELF}} = E\left(L_s|x, a, b\right) = E\left(\frac{k\rho}{1+\rho}|x, a, b\right)$$

$$= \int_0^\infty \frac{k\rho}{1+\rho}\, p_2(\rho|x, a, b)d\rho$$

$$= \frac{k}{\Gamma(a+y)U\ (a+y, a+y-nk+1, b)} \int_0^\infty e^{-b\rho} \rho^{a+y+1-1} (1 + \rho)^{-nk-1} d\rho$$

$$= k\frac{\Gamma(a+y+1)U\ (a+y+1, a+y-nk+1, b)}{\Gamma(a+y)U\ (a+y, a+y-nk+1, b)}$$

4.2 Estimation of ρ $and\ L_s$ under the PLF:

Bayesian estimate of ρ denoted by $\hat{\rho}^{GP}_{PLF}$ for a random sample of size n, $x = \left\{x_1, x_2, ..., x_n\right\}$, is given by

$$\hat{\rho}^{GP}_{PLF} = \left[E\left(\rho^2|x, a, b\right)\right]^{\frac{1}{2}}$$

$$= \left[\int_0^\infty \rho^2\, p_2(\rho|x, a, b)d\rho\right]^{\frac{1}{2}}$$

$$= \left[\frac{\Gamma(a+y+2)U\ (a+y+2, a+y-nk+3, b)}{\Gamma(a+y)U\ (a+y, a+y-nk+1, b)}\right]^{\frac{1}{2}}$$

with the associated posterior risk as follows

$$R_2\left(\hat{\rho}^{GP}_{PLF}, \rho|x, a, b\right) = 2\left[\hat{\rho}^{GP}_{PLF} - \hat{\rho}^{GP}_{SELF}\right]$$

Bayesian estimate of expected number of customers in the system L_s, denoted by $\hat{L}^{GP}_{s_{PLF}}$ is given by

$$\hat{L}^{GP}_{s_{PLF}} = \left[E\left(L_s^{\,2}|x,a,b\right)\right]^{\frac{1}{2}}$$

$$= \left[E\left(\left(\frac{k\rho}{1+\rho}\right)^2|x,a,b\right)\right]^{\frac{1}{2}}$$

$$= \left[\int_0^\infty \left(\frac{k\rho}{1+\rho}\right)^2 p_2(\rho|x,a,b)d\rho\right]^{\frac{1}{2}}$$

$$= k\left[\frac{\Gamma(a+y+2)\mathrm{U}\ (a+y+2,a+y-nk+1,b)}{\Gamma(a+y)\mathrm{U}\ (a+y,a+y-nk+1,b)}\right]^{\frac{1}{2}}$$

5. Jeffreys Prior:

When prior information is available, it is similar to adding observations to the sample size, which reduces the posterior risk of Bayesian estimate. In cases where experimenters lack of prior information about ρ, they can use Jeffreys prior (Jeffreys 1998), defined as

$$P(\theta) \propto [I(\theta)]^{\frac{1}{2}}$$

where $I(\theta)$ is the Fisher information matrix.

$$I(\theta) = -\ E\left(\frac{\partial^2 logP(x|\theta)}{\partial\theta^2}\right)$$

From the probability mass function given in equation (1), we have

$$logP(x|\rho) = log\left(\frac{k}{x}\right) + xlog\left(\frac{\rho}{1+\rho}\right) + (k\ -\ x)log\left(\frac{1}{1+\rho}\right)$$

The first and second derivatives of $logP(x|\rho)$ with respect to ρ are

$$\frac{\partial}{\partial\rho}logP(x|\rho) = x\left(\frac{1}{\rho} - \frac{1}{1+\rho}\right) + (k\ -\ x)\left(-\ \frac{1}{1+\rho}\right)$$

$$\frac{\partial^2}{\partial\rho^2}logP(x|\rho) = x\left(-\ \frac{1}{\rho^2} + \frac{1}{(1+\rho)^2}\right) + (k\ -\ x)\left(\frac{1}{(1+\rho)^2}\right)$$

Using the fact that $E(x|\rho) = k\frac{\rho}{1+\rho}$, we get the Fishers information matrix with respect to ρ as

$$I(\rho) = -\ E(x)\left(-\ \frac{1}{\rho^2} + \frac{1}{(1+\rho)^2}\right) + (k\ +\ E(x))\left(\frac{1}{(1+\rho)^2}\right)$$

$$= \frac{k}{\rho(1+\rho)^2}$$

$$= k\rho^{-1}(1\ +\ \rho)^{-2}$$

Thus, the Jeffreys prior is given by

$$q_3(\rho) \propto [I(\rho)]^{\frac{1}{2}} = \rho^{-\frac{1}{2}}(1\ +\ \rho)^{-1},\ \rho > 0 \qquad (5)$$

which is an improper distribution.

For a random sample of size n, $x = \{x_1, x_2, ..., x_n\}$, the posterior distribution of ρ for the prior $q_3(\rho)$, from equation (5), is calculated as

$$p_3(\rho|x) = \frac{q_3\left(\rho|\frac{1}{2},\frac{1}{2}\right)\times L(\rho,x)}{\int_0^\infty q_3\left(\rho|\frac{1}{2},\frac{1}{2}\right)\times L(\rho,x)d\rho}$$

$$= \frac{1}{\beta\left(y+\frac{1}{2},nk-y+\frac{1}{2}\right)}\frac{\rho^{y-\frac{1}{2}}}{(1+\rho)^{nk+1}}$$

where $y = \sum_{i=1}^{n} x_i$

5.1 Estimation of ρ *and* L_s under the SELF:

Bayesian estimate of ρ denoted by $\hat{\rho}^{JP}_{SELF}$ for a random sample of size n, $x = \{x_1, x_2, ..., x_n\}$, is given by

$$\hat{\rho}^{JP}_{SELF} = E\left(\rho|x,\frac{1}{2},\frac{1}{2}\right)$$

$$= \int_0^\infty \rho\, p_1\left(\rho|x,\frac{1}{2},\frac{1}{2}\right)d\rho$$

$$= \frac{\beta\left(y+\frac{3}{2},nk-y-\frac{1}{2}\right)}{\beta\left(y+\frac{1}{2},nk-y+\frac{1}{2}\right)}$$

$$= \frac{\left(y+\frac{1}{2}\right)}{\left(nk-y-\frac{1}{2}\right)}$$

with the associated posterior risk as follows

$$R_1\left(\hat{\rho}^{JP}_{SELF}, \rho|x,c,d\right) = V\left(\rho|x,\frac{1}{2},\frac{1}{2}\right)$$

$$= E\left(\rho^2|x,\frac{1}{2}\right),\frac{1}{2} - \left\{E\left(\rho|x,\frac{1}{2},\frac{1}{2}\right)\right\}^2$$

$$= \frac{\left(y+\frac{1}{2}\right)}{\left(nk-y-\frac{1}{2}\right)^2}\frac{\left(y+nk-\frac{1}{2}\right)}{\left(nk-y-\frac{3}{2}\right)}$$

Bayesian estimate of expected number of customers in the system L_s, denoted by $\hat{L}^{JP}_{s_{SELF}}$ is given by

$$\hat{L}^{JP}_{s_{SELF}} = E\left(L_s|x,\frac{1}{2},\frac{1}{2}\right) = E\left(\frac{k\rho}{1+\rho}|x,\frac{1}{2},\frac{1}{2}\right)$$

$$= \int_0^\infty \frac{k\rho}{1+\rho}\, p_1\left(\rho|x,\frac{1}{2},\frac{1}{2}\right)d\rho$$

$$= \frac{k}{\beta\left(y+\frac{1}{2},nk-y+\frac{1}{2}\right)}\int_0^\infty \frac{\rho^{y+\frac{3}{2}-1}}{(1+\rho)^{nk+2}}d\rho$$

$$= k\frac{\left(y+\frac{1}{2}\right)}{(nk+1)}$$

5.2 Estimation of ρ $and\ L_s$ under the PLF:

Bayesian estimate of ρ denoted by $\hat{\rho}^{JP}_{PLF}$ for a random sample of size n, $x = \{x_1, x_2, ..., x_n\}$, is given by

$$\hat{\rho}^{JP}_{PLF} = \left[E\left(\rho^2|x, \frac{1}{2}, \frac{1}{2}\right)\right]^{\frac{1}{2}}$$

$$= \left[\int_0^\infty \rho^2\, p_1\left(\rho|x, \frac{1}{2}, \frac{1}{2}\right)d\rho\right]^{\frac{1}{2}}$$

$$= \left[\frac{\beta\left(y+\frac{5}{2},\ nk-y-\frac{3}{2}\right)}{\beta\left(y+\frac{1}{2},\ nk-y+\frac{1}{2}\right)}\right]^{\frac{1}{2}}$$

$$= \left[\frac{\left(y+\frac{1}{2}\right)}{\left(nk-y-\frac{1}{2}\right)}\frac{\left(y+\frac{3}{2}\right)}{\left(nk-y-\frac{3}{2}\right)}\right]^{\frac{1}{2}}$$

with the associated posterior risk as follows

$$R_2\left(\hat{\rho}^{JP}_{PLF}, \rho|x, \frac{1}{2}, \frac{1}{2}\right) = 2\left[\hat{\rho}^{JP}_{PLF} - \hat{\rho}^{JP}_{SELF}\right]$$

Bayesian estimate of expected number of customers in the system L_s, denoted by $\hat{L}^{JP}_{s_{PLF}}$ is given by

$$\hat{L}^{JP}_{s_{PLF}} = \left[E\left(L_s^2|x, \frac{1}{2}, \frac{1}{2}\right)\right]^{\frac{1}{2}}$$

$$= \left[E\left(\left(\frac{k\rho}{1+\rho}\right)^2|x, \frac{1}{2}, \frac{1}{2}\right)\right]^{\frac{1}{2}}$$

$$= \left[\int_0^\infty \left(\frac{k\rho}{1+\rho}\right)^2 p_1\left(\rho|x, \frac{1}{2}, \frac{1}{2}\right)d\rho\right]^{\frac{1}{2}}$$

$$= \left[k^2\frac{\beta\left(y+\frac{5}{2},\ nk-y+\frac{1}{2}\right)}{\beta\left(y+\frac{1}{2},\ nk-y+\frac{1}{2}\right)}\right]^{\frac{1}{2}}$$

$$= k\left[\frac{\left(y+\frac{1}{2}\right)\left(y+\frac{3}{2}\right)}{(nk+1)(nk+2)}\right]^{\frac{1}{2}}$$

6. Credible Interval:

Suppose that we want to compute the probability that the parameter φ falls within the interval (c, d) given that $f(\varphi|x)$ represents the posterior distribution. This is given by

$$P(\varphi\in(c, d))(x) = \{\int_c^d f(\varphi|x)d\varphi,\ \ if\ \varphi\ is\ continuous\ \ \sum_{c<\varphi<d} f(\varphi|x),\ \ if\ \varphi\ is\ discrete$$

This probability represents our belief that $\varphi\in(c, d)$ based on the sample and prior information. Our goal in this section is to determine a credible interval for ρ, which is an interval $\left[\gamma_1, \gamma_2\right]$ derived from the posterior distribution and encompasses a specified fraction $(1 - \delta)$ of the degree of belief, that is

$$\int_{\gamma_1}^{\gamma_2} P(\rho|x)d\rho = 1 - \delta \qquad (6)$$

The shortest credible interval can be obtained by minimising the length of the interval $\gamma = \gamma_2 - \gamma_1$ subject to constraint (6), which gives us

$$P\left(\gamma_1|x\right) = P\left(\gamma_2|x\right) \qquad (7)$$

That is an interval $\left[\gamma_1, \gamma_2\right]$, which is simultaneously satisfies the conditions (6) & (7), is called the shortest $(1 - \delta)$ credible interval. For the sake of argument, let's consider the following equal-tailed $(1 - \delta)$ credible interval for ρ instead, as it is significantly earlier to obtain and just as useful,

$$\int_{-\infty}^{\gamma_1} P(\rho|x)d\rho = \int_{\gamma_2}^{\infty} P(\rho|x)d\rho = \frac{\delta}{2} \qquad (8)$$

Thus, we derive the following equal-tailed credible intervals for ρ from the posterior distributions using the beta prior second kind $q_1(\rho|c, d)$, gamma prior $q_2(\rho|a, b)$, and Jeffreys prior $q_3(\rho)$ respectively.

$$\int_{0}^{\gamma_1} p_1(\rho|x, c, d)d\rho = \int_{\gamma_2}^{\infty} p_1(\rho|x, c, d)d\rho = \frac{\delta}{2}$$

$$\int_{0}^{\gamma_1} p_2(\rho|x, a, b)d\rho = \int_{\gamma_2}^{\infty} p_2(\rho|x, a, b)d\rho = \frac{\delta}{2}$$

and $\int_{0}^{\gamma_1} p_3\left(\rho|x, \frac{1}{2}, \frac{1}{2}\right)d\rho = \int_{\gamma_2}^{\infty} p_3\left(\rho|x, \frac{1}{2}, \frac{1}{2}\right)d\rho = \frac{\delta}{2}$

The posterior distributions $p_1(\rho|x, c, d)$, $p_3\left(\rho|x, \frac{1}{2}, \frac{1}{2}\right)$ follows beta distributions second kind and $p_2(\rho|x, a, b)$ follows Gaussian hypergeometric distributions i.e., Kummer's confluent hypergeometric function second kind, the values of γ_1 *and* γ_2 can be obtained by (Mazumdar & Bhattacharjee, 1973) or by using statistical software such as R (R core Team 2023), Python, Minitab, MATLAB, Excel (2023), etc.

7. Predictive Distribution:

The aim of the statistical analysis is to understand and interpret the observed data as well as to make inferential statements about unknown parameters of the model to inform the client about what is likely to happen if the experiment is repeated. In many situations, the client may be more interested in making inferences about future unobserved data than the present sample information. Any inferential problem whose solution relies on a future event is a problem of statistical prediction.

Let $x = \{x_1, x_2, ..., x_n\}$ be given random sample of size n and let we are interested in predicting a future observation z on the basis of this information, i.e., $g(z|x)$, then, the posterior predictive distribution of a future observation is defined as the posterior expectation of $f(z|\varphi)$ i.e.

$$g(z|x) = \int f(z|\varphi)p(\varphi|x)d\varphi$$

where $p(\varphi|x)$ is the posterior density function of φ given x.

It is important to note that $g(z|x)$ incorporates both the sample information and prior information reflected on $p(\varphi|x)$.

Thus, the predictive distribution for the probability mass function given by equation (1) and the beta prior $q_1(\rho|c, d)$, equation (2) is given by

$$P_1(Z = z|x) = \int_0^\infty p(Z = z)p_1(\rho|x, c, d)d\rho$$

$$= \int_0^\infty \binom{k}{z}\left(\frac{\rho}{1+\rho}\right)^z\left(\frac{1}{1+\rho}\right)^{k-z}\frac{1}{\beta(c+y,d+nk-y)}\frac{\rho^{c+y-1}}{(1+\rho)^{c+d+nk}}d\rho$$

$$= \binom{k}{z}\frac{\beta(c+y+z,d+nk+k-y-z)}{\beta(c+y,d+nk-y)}, \ z = 0, 1, 2, ...$$

with the gamma prior $q_2(\rho|a, b)$ the predictive distribution is given by

$$P_2(Z = z|x) = \int_0^\infty p(Z = z)p_2(\rho|x, a, b)d\rho$$

$$= \binom{k}{z}\frac{\Gamma(a+y+z)\mathrm{U}\ (a+y+z,a+y+z-nk-k+1,b)}{\Gamma(a+y)\mathrm{U}\ (a+y,a+y-nk+1,b)}, \ z = 0, 1, 2, ...$$

with the Jeffrey's prior $q_3\left(\rho|\frac{1}{2},\frac{1}{2}\right)$ the predictive distribution is given by

$$P_3(Z = z|x) = \int_0^\infty p(Z = z)p_3\left(\rho|x,\frac{1}{2},\frac{1}{2}\right)d\rho$$

$$= \binom{k}{z}\frac{\beta\left(y+z+\frac{1}{2},nk+k-y-z+\frac{1}{2}\right)}{\beta\left(y+\frac{1}{2},nk-y+\frac{1}{2}\right)}, \ z = 0, 1, 2, ...$$

For a given data suppose we have to compare different models from a set of models, then the predictive distribution plays an important role in selecting and comparing two competing Bayesian model (Gamerman and Lopes, 2006). The posterior probabilities of the prior probabilities of the model z_i in relation to the model z_j are defined as $P_i(Z = z|x)/P_j(Z = z|x)$, which is the ratio of the prior probabilities of model z_i and z_j which is known as Bayes factor and is given by

$$B_{ij} = \frac{f(z|z_i)}{f(z|z_j)} => B_{ij} = \frac{P_i(Z=z|x)}{P_j(Z=z|x)}, \ z = 0, 1, ...$$

To take a decision between models i and j, we suggested then rule given by Jeffrey's (1998), which is stated as follows:

"There is decisive, strong, or substantial evidence against j when the Bayes B_{ij} is greater than 100, is between 10 and 100, or in between 3 and 10, respectively".

8. Simulation study and discussion

Here, we conducted simulation studies to illustrate the proposed process numerically. The following steps describe our approach:

1. For different sample sizes n=20, 50, 100, k=10, 50 and different values of ρ = 0.5, 0.7, 0.9 calculates the estimated value of ρ under ML techniques and for different choices of (c, d) = {(2,2), (3,3) for Beta prior, and (a, b) = {(2,2), (3,3)} for gamma prior and Jeffreys prior by using R packages and represented in table1.
2. Predictive probabilities are obtained for different choices of y = 0, 1, 2, 5, 7, 10 and for different values of n=20, 50, ρ = 0.4, 0.7 and (c, d) = {(2,2), (3,3) and (a, b) = {(2,2), (3,3)} and represented in table2.
3. To analyse the estimators of $\rho\, and\, L_s$, we generated random samples for different values of $\rho\in\{0.5,\ 0.7,\ 0.9\}$, sample $sizes\, n\in\{20,\ 50,\ 100\}$ and $k\in\{10,\ 50\}$. The average estimate and root mean square errors (RMSEs) over 100 Monte Carlo replications were computed for the point.

Table 1 reports, for n ∈ {20, 50, 100}, k ∈ {10, 50} and ρ ∈ {0.5, 0.7, 0.9}, the classical estimate together with the Bayes estimates under the Beta prior (c, d) ∈ {(2,2), (3,3)}, the Gamma prior (a, b) ∈ {(2,2), (3,3)} and Jeffreys' prior.

Table 1. Classical (ML) and Bayesian point estimates of ρ under SELF for the beta priors (2,2), (3,3), gamma priors (2,2), (3,3) and Jeffreys prior (JP)

n	k	ρ	$\hat{\rho}$ ML	$\hat{\rho}$ Beta(2,2)	$\hat{\rho}$ Beta(3,3)	$\hat{\rho}\,\Gamma(2,2)$	$\hat{\rho}\,\Gamma(3,3)$	$\hat{\rho}$ JP
20	10	0.5	0.4925	0.5147	0.5217	0.5217	0.5320	0.5038
50	10	0.5	0.5015	0.5060	0.5075	0.5075	0.5097	0.5038
100	10	0.5	0.4999	0.5001	0.5002	0.5002	0.5003	0.5000
20	50	0.5	0.5015	0.5060	0.5075	0.5075	0.5097	0.5038
50	50	0.5	0.4997	0.5006	0.5009	0.5009	0.5013	0.5002
100	50	0.5	0.5000	0.5001	0.5001	0.5001	0.5001	0.5000
20	10	0.7	0.6949	0.7167	0.7213	0.7227	0.7300	0.7094
50	10	0.7	0.7007	0.7051	0.7061	0.7065	0.7081	0.7036

n	k	ρ	$\hat{\rho}$ ML	$\hat{\rho}$ Beta(2,2)	$\hat{\rho}$ Beta(3,3)	$\hat{\rho}$ Γ(2,2)	$\hat{\rho}$ Γ(3,3)	$\hat{\rho}$ JP
100	10	0.7	0.7001	0.7003	0.7004	0.7004	0.7005	0.7002
20	50	0.7	0.7007	0.7051	0.7061	0.7065	0.7081	0.7036
50	50	0.7	0.6995	0.7004	0.7006	0.7007	0.7010	0.7001
100	50	0.7	0.7000	0.7000	0.7000	0.7000	0.7001	0.7000
20	10	0.9	0.8868	0.9074	0.9091	0.9093	0.9118	0.9048
50	10	0.9	0.9011	0.9053	0.9057	0.9059	0.9065	0.9048
100	10	0.9	0.9001	0.9003	0.9003	0.9003	0.9003	0.9002
20	50	0.9	0.9011	0.9053	0.9057	0.9059	0.9065	0.9048
50	50	0.9	0.8997	0.9005	0.9006	0.9007	0.9008	0.9004
100	50	0.9	0.9000	0.9000	0.9000	0.9000	0.9000	0.9000

All estimators converge to the true ρ as n or k increases. In small samples the informative priors—especially Beta (3,3) and Gamma (3,3)—sit above ρ because their prior mean exceeds it, whereas Jeffreys and the classical estimator agree closely; the gap between all methods essentially vanishes by n = 100 (k = 10) or already at n = 50 (k = 50).

Figure 1 shows the three posteriors concentrating tightly about the true value.

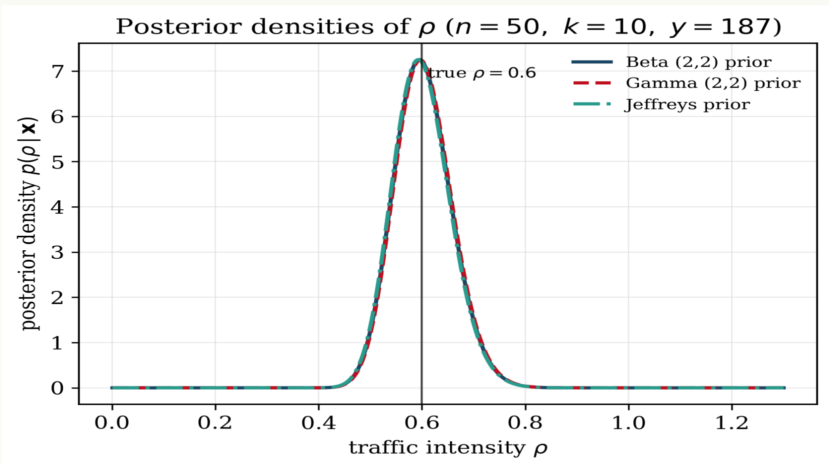

Figure 1 ***Posterior densities of ρ under the three priors for a single sample (n = 50, k = 10).***

Table 2. ***Posterior predictive probabilities P(Z=z|x) for z ∈ {0,1,2,5,7,10} (k = 10) under the beta and gamma priors.***

n	ρ	prior	z=0	z=1	z=2	z=5	z=7	z=10
20	0.4	Beta(2,2)	0.0359	0.1375	0.2426	0.0940	0.0084	0.0000
20	0.4	Beta(3,3)	0.0349	0.1350	0.2406	0.0959	0.0088	0.0000
20	0.4	Gamma(2,2)	0.0351	0.1355	0.2410	0.0956	0.0087	0.0000
20	0.4	Gamma(3,3)	0.0337	0.1320	0.2381	0.0982	0.0092	0.0000
50	0.4	Beta(2,2)	0.0348	0.1373	0.2457	0.0917	0.0075	0.0000
50	0.4	Beta(3,3)	0.0344	0.1362	0.2449	0.0925	0.0077	0.0000
50	0.4	Gamma(2,2)	0.0345	0.1364	0.2450	0.0924	0.0076	0.0000
50	0.4	Gamma(3,3)	0.0340	0.1350	0.2438	0.0935	0.0078	0.0000
20	0.7	Beta(2,2)	0.0058	0.0375	0.1119	0.2063	0.0516	0.0002
20	0.7	Beta(3,3)	0.0057	0.0370	0.1111	0.2070	0.0521	0.0002
20	0.7	Gamma(2,2)	0.0056	0.0369	0.1108	0.2072	0.0523	0.0002
20	0.7	Gamma(3,3)	0.0055	0.0362	0.1095	0.2083	0.0530	0.0002
50	0.7	Beta(2,2)	0.0052	0.0354	0.1096	0.2092	0.0506	0.0002
50	0.7	Beta(3,3)	0.0052	0.0353	0.1092	0.2095	0.0508	0.0002
50	0.7	Gamma(2,2)	0.0052	0.0352	0.1091	0.2096	0.0509	0.0002
50	0.7	Gamma(3,3)	0.0051	0.0349	0.1086	0.2100	0.0512	0.0002

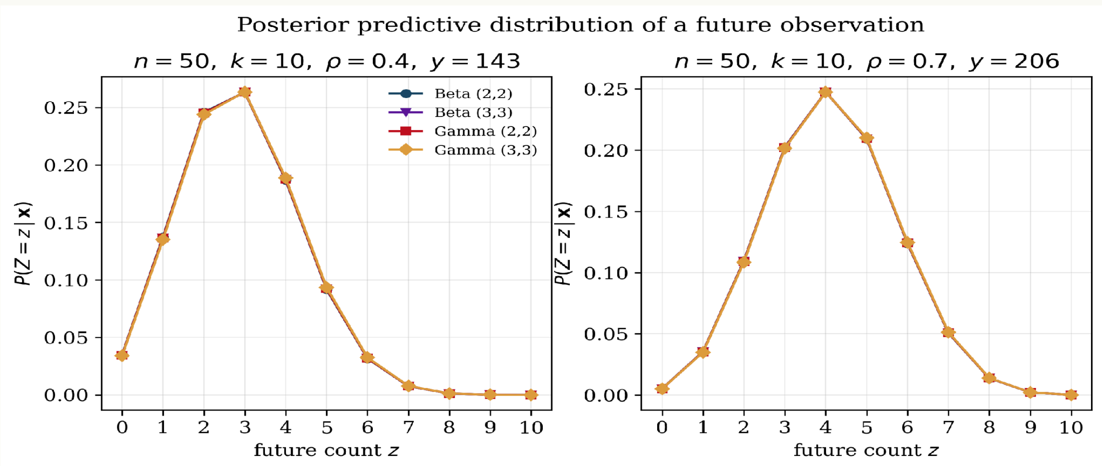

Figure 2. ***Posterior predictive distribution of a future observation for ρ = 0.4 and 0.7 (n = 50, k = 10).***

Posterior predictive probabilities $P(Z = z \mid x)$ of a future observation were obtained for $z \in \{0, 1, 2, 5, 7, 10\}$ $(k = 10)$, $n \in \{20, 50\}$ and $\rho \in \{0.4, 0.7\}$ under the beta priors (2,2),(3,3) and gamma priors (2,2),(3,3). The predictive mass is almost identical across priors, Figure 5.9.2, shows the corresponding Bayes factors are close to unity and prediction is robust to the prior once the sample is informative.

Table 3 Average Bayes estimate of ρ with RMSE (in parentheses) under SELF, 1,000 replications.

ρ	n	k	Beta (2,2)	Gamma (2,2)	Jeffreys
0.5	20	10	0.5290 (0.1166)	0.5355 (0.1183)	0.5184 (0.1163)
0.5	50	10	0.5062 (0.0497)	0.5076 (0.0499)	0.5040 (0.0496)
0.5	100	10	0.5001 (0.0105)	0.5002 (0.0105)	0.5000 (0.0105)
0.7	20	10	0.7324 (0.1455)	0.7373 (0.1442)	0.7258 (0.1474)
0.7	50	10	0.7096 (0.0626)	0.7110 (0.0627)	0.7082 (0.0626)
0.7	100	10	0.7001 (0.0144)	0.7001 (0.0144)	0.7000 (0.0144)
0.9	20	10	0.9352 (0.1826)	0.9345 (0.1755)	0.9338 (0.1875)
0.9	50	10	0.9047 (0.0811)	0.9053 (0.0806)	0.9042 (0.0815)
0.9	100	10	0.9006 (0.0179)	0.9006 (0.0179)	0.9006 (0.0179)
0.5	20	50	0.5055 (0.0476)	0.5070 (0.0479)	0.5033 (0.0476)
0.5	50	50	0.5010 (0.0214)	0.5013 (0.0214)	0.5006 (0.0213)
0.5	100	50	0.5002 (0.0047)	0.5002 (0.0047)	0.5001 (0.0047)
0.7	20	50	0.7052 (0.0647)	0.7065 (0.0646)	0.7037 (0.0648)
0.7	50	50	0.6992 (0.0289)	0.6995 (0.0289)	0.6989 (0.0290)
0.7	100	50	0.7004 (0.0064)	0.7004 (0.0064)	0.7004 (0.0064)
0.9	20	50	0.9077 (0.0813)	0.9082 (0.0808)	0.9072 (0.0817)
0.9	50	50	0.9019 (0.0362)	0.9020 (0.0362)	0.9018 (0.0363)
0.9	100	50	0.8999 (0.0082)	0.8999 (0.0082)	0.8999 (0.0082)

Table 4 Average Bayes estimate of Ls with RMSE (in parentheses) under SELF, 100 replications.

ρ	n	k	Beta (2,2)	Gamma (2,2)	Jeffreys
0.5	20	10	3.3918 (0.4729)	3.4204 (0.4737)	3.3441 (0.4834)

ρ	n	k	Beta (2,2)	Gamma (2,2)	Jeffreys
0.5	50	10	3.3470 (0.2159)	3.3534 (0.2161)	3.3371 (0.2168)
0.5	100	10	3.3330 (0.0467)	3.3333 (0.0467)	3.3325 (0.0467)
0.7	20	10	4.1496 (0.4619)	4.1681 (0.4543)	4.1244 (0.4745)
0.7	50	10	4.1350 (0.2113)	4.1396 (0.2109)	4.1298 (0.2122)
0.7	100	10	4.1170 (0.0500)	4.1173 (0.0500)	4.1168 (0.0500)
0.9	20	10	4.7431 (0.4688)	4.7459 (0.4534)	4.7354 (0.4826)
0.9	50	10	4.7311 (0.2221)	4.7327 (0.2207)	4.7295 (0.2235)
0.9	100	10	4.7376 (0.0494)	4.7377 (0.0494)	4.7375 (0.0494)
0.5	20	50	16.7236 (1.0371)	16.7560 (1.0382)	16.6741 (1.0418)
0.5	50	50	16.6761 (0.4731)	16.6827 (0.4732)	16.6661 (0.4736)
0.5	100	50	16.6698 (0.1044)	16.6701 (0.1044)	16.6693 (0.1044)
0.7	20	50	20.5946 (1.1035)	20.6181 (1.0999)	20.5683 (1.1103)
0.7	50	50	20.5573 (0.5005)	20.5622 (0.4998)	20.5519 (0.5014)
0.7	100	50	20.5949 (0.1102)	20.5952 (0.1102)	20.5947 (0.1101)
0.9	20	50	23.6961 (1.1032)	23.7041 (1.0960)	23.6883 (1.1097)
0.9	50	50	23.6917 (0.4992)	23.6935 (0.4985)	23.6901 (0.4997)
0.9	100	50	23.6818 (0.1131)	23.6819 (0.1131)	23.6818 (0.1131)

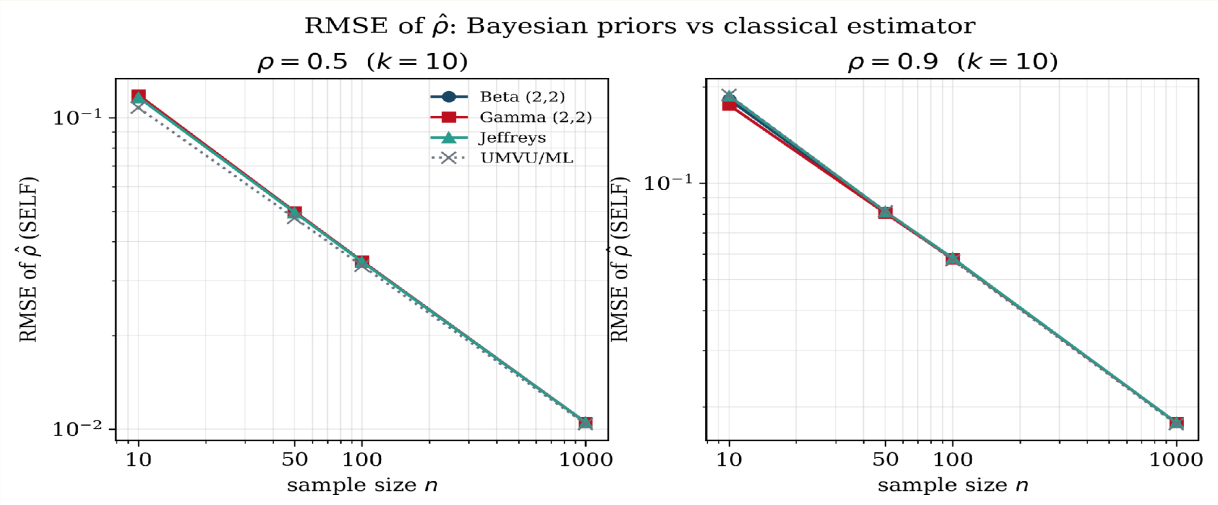

Figure 3 ***RMSE of ρ̂ (SELF) vs sample size n (log–log) for ρ = 0.5 and 0.9 at k = 10, comparing the three priors with the classical estimator.***

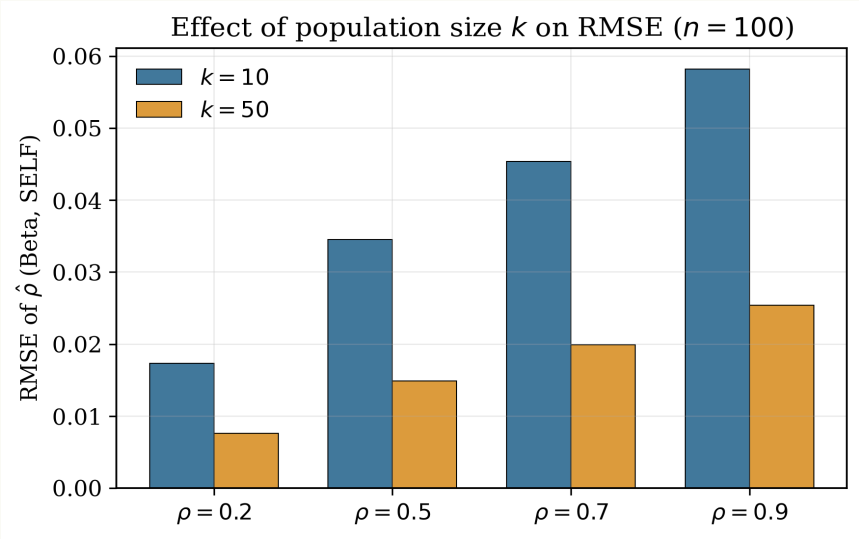

Figure 4 ***Effect of population size k on the RMSE of ρ̂ (beta prior, n = 100).***

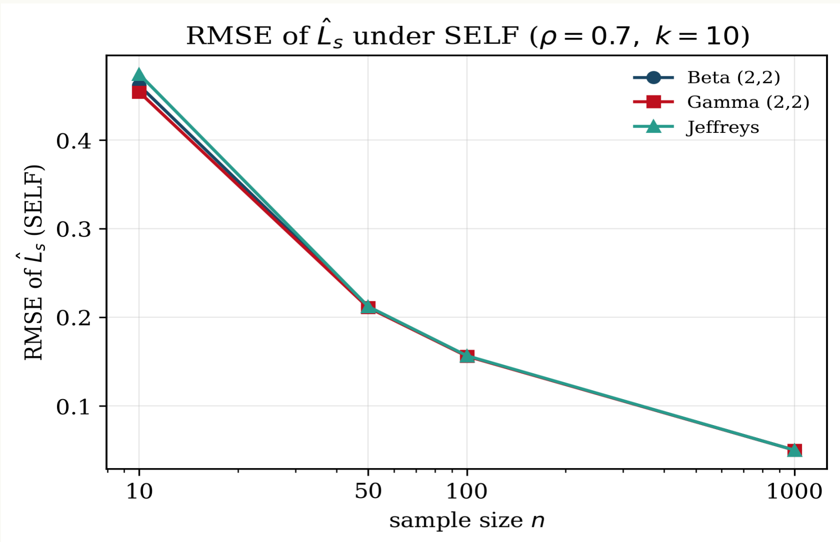

Figure 5 ***RMSE of Ls (SELF) vs n for the three priors (ρ = 0.7, k = 10).***

Random samples were generated for $\rho \in \{0.2, 0.5, 0.7, 0.9\}$, $n \in \{20, 50, 100\}$ and $k \in \{10, 50\}$. Over 1,000 Monte Carlo replications, the average estimate and RMSE (in parentheses) of ρ and of the expected system size $L_s = k\rho/(1+\rho)$ were computed under the beta (2,2), gamma (2,2) and Jeffreys priors. The RMSE decreases steadily with both n and k, and the three priors are practically indistinguishable for $n \geq 100$ (Figures 3–5). The Bayes estimators match the classical estimator for large n but are slightly more accurate in small samples (Figure 3).

Table 5 RMSE of ρ̂ under SELF (S) and PLF (P) at k = 10 for the beta (2,2), gamma (2,2) and Jeffreys (JP) priors.

ρ	n	Beta S	Beta P	Gamma (S)	Gamma (P)	JP S	JP P
0.5	20	0.1166	0.1216	0.1183	0.1237	0.1163	0.1205
0.5	50	0.0497	0.0502	0.0499	0.0505	0.0496	0.0500
0.5	100	0.0105	0.0105	0.0105	0.0105	0.0105	0.0105
0.7	20	0.1455	0.1518	0.1442	0.1507	0.1474	0.1532
0.7	50	0.0626	0.0633	0.0627	0.0634	0.0626	0.0633
0.7	100	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144
0.9	20	0.1826	0.1904	0.1755	0.1827	0.1875	0.1955
0.9	50	0.0811	0.0817	0.0806	0.0813	0.0815	0.0821
0.9	100	0.0179	0.0179	0.0179	0.0179	0.0179	0.0179

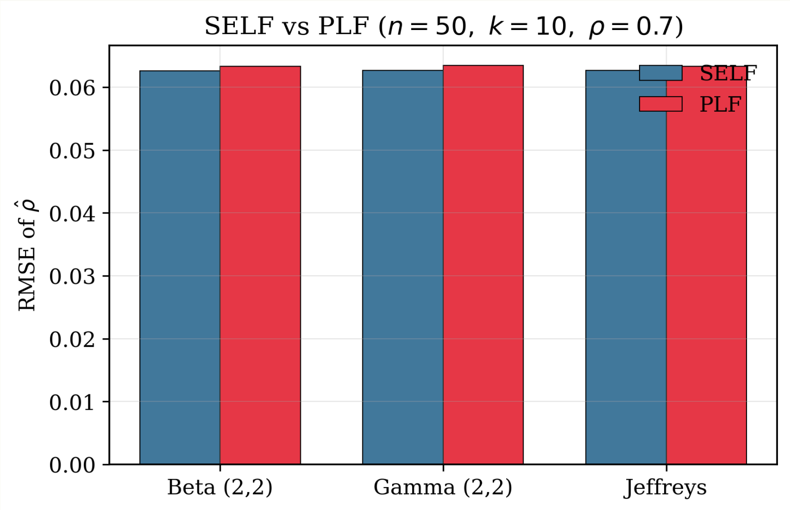

Figure 6 ***RMSE of $\hat{\rho}$ under SELF versus PLF for the three priors ($n = 50$, $k = 10$, $\rho = 0.7$)***.

Table 5 and Figure 6 compare the two loss functions. The precautionary loss returns uniformly larger, more conservative estimates of ρ than the squared-error loss—guarding against under-estimation of congestion—at the cost of a small increase in RMSE. The difference narrows as n grows and is negligible for large samples, so PLF is attractive when under-provisioning the repair facility is costly, while SELF is preferable when a symmetric, minimum-variance criterion is desired.

Across all studies the picture is consistent. (i) Every estimator is consistent: bias and RMSE fall steadily with n and k. (ii) The choice of prior matters only in small samples—informative beta/gamma priors trade a little bias for efficiency, Jeffreys stays close to the likelihood, and all three converge to the classical ML estimator for moderate n. (iii) Estimation is harder as $\rho \to 1$, but a larger finite source k sharply reduces the error. (iv) Prediction is robust to the prior. (v) The precautionary loss gives conservative estimates useful when under-estimation is costly, whereas the squared-error loss is the natural symmetric choice. In practice, use Jeffreys' prior when prior information is scarce, an informative beta or gamma prior when reliable knowledge exists, and PLF when guarding against under-provisioning.

9. Conclusions

This paper has developed a comprehensive Bayesian framework for estimating the traffic intensity ρ and the expected number of customers Ls in the machine-interference model, in which the number of failed units follows a binomial distribution with success probability ρ/(1+ρ). Bayes estimators and their posterior risks were derived under four prior distributions—a natural-conjugate beta prior of the second kind, a non-conjugate gamma prior, Jeffreys' prior and the asymptotically locally invariant prior—and under two loss functions, the symmetric squared-error loss and the asymmetric precautionary loss. Equal-tailed credible intervals, posterior predictive distributions and the corresponding Bayes factors were also obtained, providing a complete toolkit for point and interval estimation as well as model comparison.

The Monte Carlo study confirms several practically important patterns. First, every Bayes estimator is consistent: the average estimates approach the true value and the root-mean-square error declines steadily as the sample size increases (Table 3, Figure 2). Second, at small sample sizes the informative beta and gamma priors carry a modest efficiency advantage, whereas the non-informative Jeffreys and ALI priors exhibit slightly smaller bias; the four procedures become practically indistinguishable once the sample is moderately large, and all agree closely with the maximum-likelihood estimators (Table 1). Third, the precautionary loss guards more strongly against under-estimation, yielding uniformly larger estimates than squared-error loss at the cost of a small increase in risk (Figure 3). The predictive Bayes factors remain close to unity across priors, indicating good agreement among the fitted models. These findings offer clear guidance for practitioners selecting priors and loss functions in finite-source queueing applications, and point to natural extensions to multi-server and non-exponential systems.

References

Acharya, S. K. (1999). On normal approximation for maximum likelihood estimation from single server queues. *Queueing systems*, 31, 207-216.

Almeida, M. A., & Cruz, F. R. (2018). A note on Bayesian estimation of traffic intensity in single-server Markovian queues. *Communications in Statistics- Simulation and Computation*, 47(9), 2577-2586. Armero, C., & Bayarri, M. J. (1994a). Bayesian prediction in M/M/1 queues. *Queueing Systems*, 15, 401-417.

Armero C, Bayarri MJ (1994b). Prior assessments for prediction in queues. *Statistician*, 43(1), 139-153.

Armero C, Bayarri MJ (1999). Dealing with uncertainties in queues and networks of queues: a Bayesian approach. In: Ghosh S (ed). *Multivariate analysis, design of experiments and survey sampling.* Marcel Dekker, New York, 579-608.

Basak, A., & Choudhury, A. (2021). Bayesian inference and prediction in single server M/M/1 queuing model based on queue length. *Communications in Statistics-Simulation and Computation*, 50(6), 1576-1588.

Basawa, I. V., & Prabhu, N. U. (1988). Large sample inference from single server queues. *Queueing Systems*, 3(4), 289-304.

Basawa, I. V., Lund, R., & Bhat, U. N. (1997). Estimating function methods of inference for queueing parameters. *Lecture Notes-Monograph Series*, 269-284.

Bhat, U. N., & Rao, S. S. (1972). A statistical technique for the control of traffic intensity in the queuing systems M/G/1 and GI/M/1. *Operations Research*, 20(5), 955-966.

Bhat, U. N., & Rao, S. S. (1987). Statistical analysis of queueing systems. *Queueing Systems*, 1(3), 217-247.

Bhat, U. N., Miller, G. K., & Rao, S. S. (1997). Statistical analysis of queueing systems. In J. H. Dshalalow (Ed.), *Frontiers in queueing: Models and applications in science and engineering* (pp. 351–393). New York: CRC Press.

Chen, J.J. & Novick, M.R. (1984) Bayesian Analysis for Binomial Models with Generalized Beta Distributions. Journal of Educational Statistics, Vol.9, No. 2, 163-175

Choudhury, A., & Basak, A. (2018). Statistical inference on traffic intensity in an M/M/1 queueing system. *International Journal of Management Science and Engineering Management*, 13(4), 274-279.

Chowdhury, S., & Mukherjee, S. P. (2011). Estimation of waiting time distribution in an M/M/1 Queue. *Opsearch*, 48(4), 306-317.

Chowdhury, S., & Mukherjee, S. P. (2013). Estimation of traffic intensity based on queue length in a single M/M/1 queue. *Communications in Statistics-theory and Methods*, 42(13), 2376-2390.

Clarke, A. B. (1957). Maximum likelihood estimates in a simple queue. *The Annals of Mathematical Statistics*, 28(4), 1036-1040.

Cruz, F. R., Quinino, R. C., & Ho, L. L. (2017). Bayesian estimation of traffic intensity based on queue length in a multi-server M/M/s queue. *Communications in Statistics-Simulation and Computation*, 46(9): 7319-7331.

Cruz, F. R., Almeida, M. A., D'Angelo, M. F., & van Woensel, T. (2018).Traffic intensity estimation in finite Markovian queueing systems. *Mathematical Problems in Engineering*, Vol.2018, Hindawi.

Das, B. and Choudhury, A. (2021) Mle and Bayes Estimator of Utilization Factor in Power Supply Queuing Model in Turkish Journal of Computer and Mathematics Education, Vol.12, No.9, 2092-2102,

Dutta, K., & Choudhury, A. (2023). Frequentist inference on traffic intensity of M/M/1 queuing system. *Operation Research and Decisions,33*(1), 22-34.

Exton, H. (1976). Multiple hypergeometric functions and applications. Chichester: Ellis Howard.

Gross, D., & Harris, C.M. (2008). *Fundamentals of Queuing Theory*, 3rd ed. (Reprinted), Wiley India (P.) Ltd., Daryaganj, New Delhi.

Gun, A.M., Gupta, M.K. & Dasgupta, B. (2005). An Outline of Statistical Theory. Vol-II, 3rd edition, World Press.

Mazumdar, K.L. & Bhattacharjee, G.P. (1973), Algorithm AS 63: The Incomplete Beta Integral. Journal of Royal Statistical Society. Series C (Applied Statistics). Vol.22, N0.3, 409-411

McGrath, M. F., & Singpurwalla, N. D. (1987). A subjective Bayesian approach to the theory of queues II—Inference and information in M/M/1 queues. *Queueing Systems*, *1*, 335-353

Muddapur, M.V. (1972). Bayesian estimates of parameters in some queueing models. Annals of the Institute of Statistical Mathematics 24(1):327–31. doi:10.1007/BF02479762.

Sharma, K. K., & Kumar, V. (1999). Inferences on M/M/1:(∞; FIFO) queue system. *Opsearch*, 36(1), 26-34.

Shortle, J. F., Thompson, J. M., Gross, D., & Harris, C. M. (2018). Fundamentals of Queuing Theory. h