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Beyond the Autonomous Consumer: A Theory of Proxy-Influence and Delegated Utility in Consumer Choice

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Abstract

The study challenges the foundational axiom of consumer theory that the agent holding purchasing power is identical to the agent forming preferences. We introduce the Proxy-Influence Consumer (PIC), an agent with zero budget authority who systematically shapes the preferences of a Budget-Constrained Principal Consumer (BCPC). Formalizing this via a Delegated Utility Function (DUF), we derive new equilibrium conditions, the Influence Elasticity of Demand (IED), and prove that Slutsky symmetry and the Weak Axiom of Revealed Preference fail under PIC influence. Applications to fast-moving consumer goods markets rationalize child-directed advertising strategies and motivate a new welfare measure, Delegated Preference Distortion.

Keywords: Consumer Theory, Utility Maximization, Delegated Utility, Proxy-Influence Consumer, Household Economics, Revealed Preference, Marketing Strategy

JEL Codes: D10, D11, D12, D90, D91

INTRODUCTION

The canonical consumer problem presents a deceptively simple picture of economic agency: a single rational individual, endowed with income m and facing a price vector p, selects a bundle of goods x to maximize a well-behaved utility function $U(x)$ subject to $p \cdot x \leq m$. This formulation is elegant, tractable, and enormously productive. It underpins demand theory, welfare economics, general equilibrium analysis, and much of industrial organization.

Yet this elegance conceals a profound assumption, one so deeply embedded in the framework that it has rarely been stated explicitly, let alone questioned: the agent who faces the budget constraint is assumed to be identical to the agent whose preferences drive the choice. The buyer is the preferrer. The payer is the person whose utility is being maximized. In the standard model, purchasing power and preference formation are inseparably fused within a single economic individual.

This paper argues that this fusion is an empirical error that has become increasingly costly as modern market structures diverge from the conditions under which the axiom was plausible. We identify a pervasive and structurally important class of economic interaction one in which the agent who forms or modifies preferences (the Proxy-Influence Consumer, or PIC) is systematically distinct from the agent who controls the budget and executes the purchase (the Budget-Constrained Principal Consumer, or BCPC). The PIC exercises what we term delegated influence: without possessing any direct purchasing power, they reliably shift the preference ordering of the BCPC, and through that shift, determine consumption outcomes.

The archetypal instance is the child consumer. Children in modern households are economically powerless in the formal sense they hold no income, face no budget constraint, and sign no contracts. Yet the marketing industry, with its billions in annual child-directed advertising expenditure, has long understood what consumer theory has formally ignored: children are extraordinarily powerful preference agents. When a child watches a television advertisement, internalizes a brand message, and demands a specific product from a parent and when the parent adjusts their purchasing decision in response we observe a complete consumer influence chain that standard models cannot represent. The Milo and Bournvita advertising phenomenon in West African markets illustrates this sharply: both brands allocate substantial advertising budgets to child-directed creative while the purchasing decision is invariably made by an adult managing the household food budget.

The child case is important but not unique. The PIC-BCPC structure characterizes a wide range of economically significant relationships: spouses whose consumption patterns reflect household negotiation; elderly parents whose purchases are guided by adult children; patients whose medication choices are shaped by caregivers; and social media users whose preferences are continuously modulated by influencers who hold no purchasing power.

Our contributions are as follows. We formally define the PIC and BCPC and specify the conditions under which their interaction is economically significant. Second, we develop the Delegated Utility Function (DUF), a novel theoretical construct that incorporates PIC influence into the BCPC's optimization problem. Third, we derive new equilibrium conditions and comparative statics, including the Influence Elasticity of Demand (IED). Fourth, we prove that Slutsky symmetry and the Weak Axiom of Revealed Preference (WARP) fail under PIC influence. Fifth, we apply the framework to fast-moving consumer goods (FMCG) markets and derive implications for advertising strategy and consumer welfare. Sixth, we introduce Delegated Preference Distortion (DPD) as a new welfare measure for advertising regulation.

The paper proceeds as follows. Section I reviews classical consumer theory and its behavioral extensions, identifying the common assumption of preference-expenditure unity. Section II introduces the formal theory of the PIC. Section III develops the DUF and derives the extended consumer problem. Section IV presents new equilibrium conditions and the IED. Section V examines implications for classical demand theorems. Section VI provides graphical analysis. Section VII presents empirical applications. Sections

VIII–XI discuss policy implications, household networks, revised axioms, and the future research agenda. Section XII concludes.

I. CLASSICAL CONSUMER THEORY: FOUNDATIONS AND LIMITATIONS

A. The Standard Consumer Problem

Let $X \subseteq \mathbb{R}^n_+$ be the consumption set, with typical element $x = (x_1, x_2, ..., x_n)$. Let $p \in \mathbb{R}^n_{++}$ be the price vector and $m > 0$ be income. The consumer's preferences are represented by a utility function $U: X \to \mathbb{R}$ satisfying standard regularity conditions. The consumer solves:

(1) $$\max U(x) \text{ subject to } p \cdot x \leq m, x \geq 0,$$

yielding the Walrasian demand function:

(2) $$x^*(p, m) = \text{argmax} \{ U(x) : p \cdot x \leq m, x \geq 0 \}.$$

Under completeness, transitivity, continuity, local non-satiation, and strict convexity, a unique solution exists, $x^*(p, m)$ is homogeneous of degree zero in (p, m), and Walras's Law holds: $p \cdot x^*(p, m) = m$. The Indirect Utility Function $V(p, m) = U(x^*(p, m))$ is decreasing in p, increasing in m, homogeneous of degree zero, and quasi-convex in (p, m). Roy's Identity recovers demand:

(3) $$x^*_i(p, m) = -[\partial V(p,m)/\partial p_i] / [\partial V(p,m)/\partial m].$$

The Slutsky equation decomposes the price response of Marshallian demand:

(4) $$\partial x^*_i/\partial p_j = \partial h_i/\partial p_j - x^*_j \cdot \partial x^*_i/\partial m,$$

where $h(p, u)$ is Hicksian demand. The Slutsky substitution matrix S, with elements $S_{ij} = \partial h_i/\partial p_j$, is symmetric and negative semi-definite results depending critically on preferences being stable, exogenous, and belonging entirely to the purchasing agent.

B. Behavioral Extensions and Their Limitations

Behavioral economics, from Kahneman and Tversky's Prospect Theory (1979) through Thaler's mental accounting (1985), Laibson's hyperbolic discounting (1997), and the synthesis in Camerer, Loewenstein, and Rabin (2004), has enriched consumer theory by relaxing perfect rationality. However, behavioral economics preserves the foundational assumption we challenge: even in behavioral models, preferences however irrationally formed belong to the purchasing agent. A hyperbolic discounter faces her own time inconsistency. A loss-averse shopper responds to her own reference point. The locus of preference formation remains the locus of budget authority.

Household economics represents a more relevant departure. The collective model of Browning and Chiappori (1998) recognizes multiple agents whose preferences are aggregated through Pareto-efficient bargaining. Let a household consist of agents a and b with utility functions $U_a(x)$ and $Ub(x)$. The collective problem is:

(5) $$\max \mu \cdot U_a(x_a, xb) + (1-\mu) \cdot Ub(x_a, xb) \text{ subject to } p \cdot (x_a + xb) \leq m,$$

where $\mu \in [0,1]$ is the Pareto weight. This framework has yielded important insights notably Browning and Chiappori's (1998) derivation that the Slutsky matrix in collective models need not be symmetric.

Yet the collective model still assumes that all household agents are both preference-holders and potential budget authorities. Children are treated as passive recipients of parental decisions, entering the household utility function through parental altruism (Becker 1991) rather than as active preference-transmitting agents. This is the gap our theory fills.

C. Four Untenable Assumptions in Modern Markets

We identify four assumptions in classical and behavioral consumer theory that are empirically untenable in modern market environments:

- ***Assumption C1 (Preference-Expenditure Unity):*** The agent maximizing utility is identical to the agent subject to the budget constraint. No agent outside the budget constraint systematically modifies the preferences of the constrained agent.
- ***Assumption C2 (Preference Exogeneity):*** Consumer preferences are formed prior to and independently of market interactions. Advertising may inform consumers but does not systematically alter the preference ordering of one agent through influence transmitted via a second agent.
- ***Assumption C3 (Unitary Preference Authority):*** In any purchasing decision, preference authority and budgetary authority are co-held by a single decision-making unit. Even in household models, all preference-bearing agents are also, at minimum, potential budget authorities.
- ***Assumption C4 (Bilateral Market Structure):*** The relevant market interaction is bilateral: firm and consumer. Third parties affect the consumer only through direct preference modification, not through the mediation of a second consumer whose preferences they shape.

We demonstrate that all four assumptions are violated in empirically important cases, and that these violations are systematically exploited by firms. A theory that accounts for these violations generates substantively different predictions about demand, firm strategy, and welfare.

II. THEORY OF THE PROXY-INFLUENCE CONSUMER

A. Formal Definitions

- ***Definition 1 (Budget-Constrained Principal Consumer):*** A Budget-Constrained Principal Consumer is an economic agent is characterized by (i) a well-defined preference relation $\succsim_i$ over consumption set X_i; (ii) a budget endowment $m_i > 0$; and (iii) sole legal authority to execute purchase decisions within that budget. Examples: parents purchasing household goods, spouses managing shared consumption.
- ***Definition 2 (Proxy-Influence Consumer):*** A Proxy-Influence Consumer is an economic agent j characterized by (i) well-defined preferences over consumption set X_j; (ii) zero direct budget endowment: $m_j = 0$; (iii) no legal authority to execute purchase decisions independently; and (iv) a non-trivial influence capacity $\omega_{ji} > 0$ over the preference ordering of BCPC i, such that the BCPC's effective preference ordering is a function of the PIC's expressed preferences. Examples: children in a household, elderly relatives under caregiver supervision, social media influencers.
- ***Definition 3 (Influence Capacity):*** The influence capacity $\omega_{ji} \in [0,1]$ of PIC j on BCPC i is a scalar measuring the weight given by i to j's expressed preferences in forming i's effective preference ordering. $\omega_{ji} = 0$ implies no influence (the standard model); $\omega_{ji} = 1$ implies complete preference delegation.

B. The Influence Capacity Function

The influence capacity is not a fixed parameter but a function of observable and potentially manipulable variables:

(6) $$\omega_{ji} = \omega(R_{ji}, E_{ji}, A_{ji}, T_{ji}),$$

where R_{ji} is relational closeness between j and i (e.g., parent-child bond strength); E_{ji} is emotional intensity of j's expressed preferences as perceived by i; A_{ji} is accumulated exposure of i to j's preferences; and T_{ji} is third-party amplification capturing how external agents (firms, advertisers) strategically enhance j's influence on i. This specification reveals the strategic opportunity available to firms: advertising directed at the PIC raises E_{ji} and A_{ji}, thereby raising ω_{ji} and increasing the probability that the BCPC's purchasing decision favors the firm's product.

We impose the following regularity conditions on ω:

1. **Boundedness:** $\omega_{ji} \in [0,1]$ for all $(j, i, R_{ji}, E_{ji}, A_{ji}, T_{ji})$.
2. **Monotonicity:** $\partial\omega/\partial R_{ji} \geq 0$, $\partial\omega/\partial E_{ji} \geq 0$, $\partial\omega/\partial A_{ji} \geq 0$, $\partial\omega/\partial T_{ji} \geq 0$. Greater relational closeness, emotional intensity, accumulated exposure, and third-party amplification all weakly increase influence capacity.
3. **Diminishing Returns:** $\partial^2\omega/\partial A^2_{ji} \leq 0$. The marginal effect of additional exposure diminishes (saturation effects).
4. **Complementarity:** $\partial^2\omega/\partial R_{ji}\partial T_{ji} \geq 0$. Third-party amplification is more effective when relational closeness is higher.

C. A Taxonomy of PIC-BCPC Relationships

The PIC-BCPC structure appears across a wide range of economically important contexts. Table 1 provides a systematic taxonomy.

TABLE 1 TAXONOMY OF PIC-BCPC RELATIONSHIPS ACROSS MARKET CONTEXTS

PIC	BCPC	Market Domain	ω Range	Firm Strategy
Children (0–12)	Parents/Guardians	Food, beverage, toys, entertainment	0.3–0.8	Child-directed advertising
Adolescents (13–18)	Parents	Electronics, fashion, digital services	0.2–0.6	Peer/teen influencer marketing
Social media influencers	Followers (buyers)	Beauty, fashion, supplements	0.1–0.5	Sponsored content, affiliate models
Non-earning spouses	Earning spouse/household	Home goods, food, services	0.3–0.9	Household/family positioning
Elderly dependants	Adult children/caregivers	Healthcare, food, housing	0.2–0.7	Caregiver-directed marketing
Patients	Family/caregiver payers	Pharmaceuticals, devices	0.4–0.9	Direct-to-patient advertising

Pets (preference signals)	Pet owners	Pet food, accessories, vet	0.1–0.4	Anthropomorphic branding

Note: The influence capacity range (ω) is indicative and varies across household structure, cultural context, and advertising intensity.

III. THE DELEGATED UTILITY FUNCTION AND THE EXTENDED CONSUMER PROBLEM

A. Constructing the Delegated Utility Function

Let BCPC have an intrinsic utility function $U^0_i(x)$ representing preferences that i would hold in the complete absence of PIC influence. Let PIC j have a preference function $V_j(x)$ over the same consumption set. We define the Delegated Utility Function (DUF) of BCPC i under influence from PIC j as:

(7) $$DUF_{ij}(x) = (1 - \omega_{ji}) \cdot U^0_i(x) + \omega_{ji} \cdot V_j(x),$$

where $\omega_{ji} \in [0,1]$ is the influence capacity from Definition 3. The BCPC's effective utility function is a convex combination of their own intrinsic preferences (weighted by $1 - \omega_{ji}$) and the PIC's preferences (weighted by ω_{ji}). When $\omega_{ji} = 0$, $DUF_{ij} = U^0_i$ and we recover the standard model. When $\omega_{ji} = 1$, $DUF_{ij} = V_j$ and the BCPC acts purely on the PIC's preferences.

For multiple PICs e.g., a household where both children and an elder influence the parental purchasing decision we generalize to:

(8) $$DUF_i(x) = \alpha^0_i \cdot U^0_i(x) + \Sigma_j\, \alpha_{ji} \cdot V_j(x),$$

where $\alpha^0_i + \Sigma_j\, \alpha_{ji} = 1$ with $\alpha_k \geq 0$ for all k. This normalization ensures DUF_i remains a proper utility function over the consumption set.

B. The Extended Consumer Problem

The extended consumer problem for BCPC is under influence from PICs j = 1, ..., J is:

(9) $$\max DUF_i(x) = \alpha^0_i \cdot U^0_i(x) + \Sigma_j\, \alpha_{ji} \cdot V_j(x) \quad s.t. \quad p \cdot x \leq m_i,\ x \geq 0, \text{ and } \omega_{ji} = \omega(R_{ji}, E_{ji}, A_{ji}, T_{ji})\ \forall j.$$

Note three crucial features. First, only the BCPC faces a budget constraint; PIC influence enters through the objective function. Second, the effective objective function is endogenous to the market environment; firms can manipulate α_{ji} through advertising targeted at PICs (raising E_{ji} and A_{ji}). Third, the optimization is formally equivalent to a standard consumer problem with a modified utility function, permitting application of standard analytical tools.

C. Existence and Uniqueness

- ***Proposition 1 (Existence):*** If U^0_i and V_j are continuous, quasi-concave, and locally non-satiated on $X = \mathbb{R}^n_+$, and if $\alpha_k \geq 0$ with $\Sigma\alpha_k = 1$, then DUF_i is continuous, quasi-concave, and locally non-satiated on X. Therefore, the extended consumer problem (9) has a solution.

Proof. DUF_i is a convex combination of continuous functions and is therefore continuous. Quasi-concavity follows from the fact that the upper contour set of a convex combination of quasi-concave functions is convex. Local non-satiation: for any $x \in X$ and $\varepsilon > 0$, since U^0_i is locally non-satiated, $\exists x'$ with $\|x - x'\| < \varepsilon$ and $U^0_i(x') > U^0_i(x)$. Then $DUF_i(x') > DUF_i(x)$ under well-defined DUF weights. The budget set $\{x : p \cdot x \leq m_i, x \geq 0\}$ is compact and non-empty for $m_i > 0$. By the Extreme Value Theorem, a continuous function on a compact set attains its maximum. ∎

- ***Proposition 2 (Uniqueness):*** If U^0_i and all V_j are strictly quasi-concave, then DUF_i is strictly quasi-concave and the solution to the extended consumer problem is unique.

D. The Delegated Demand Function

The solution to the extended consumer problem defines the Delegated Demand Function:

(10) $$\tilde{x}(p, m_i, \omega) = \text{argmax} \{ DUF_i(x) : p \cdot x \leq m_i \},$$

where $\omega = (\omega_{ji})_j$ is the vector of influence capacities, themselves functions of structural parameters (R, E, A, T). The delegated demand function differs from the standard Walrasian demand $x^*(p, m)$ in a critical respect: it depends not only on prices and income, but also on the influence vector ω. This third argument absent from all standard demand functions is the formal source of the empirical patterns we seek to explain.

IV. EQUILIBRIUM CONDITIONS AND THE INFLUENCE ELASTICITY OF DEMAND

A. First-Order Conditions

Assuming an interior solution, the Lagrangian for the extended consumer problem is:

(11) $$L = DUF_i(x) - \lambda(p \cdot x - m_i).$$

The first-order conditions are:

(12) $$\partial DUF_i/\partial x_k = \lambda \cdot p_k \text{ for } k = 1, \ldots, n,$$

(13) $$p \cdot x = m_i.$$

Expanding equation (12) via the DUF definition:

(14) $$\alpha^0_i \cdot \partial U^0_i/\partial x_k + \Sigma_j\, \alpha_{ji} \cdot \partial V_j/\partial x_k = \lambda \cdot p_k.$$

This central equilibrium condition states that the preference-weighted marginal utility of good k across the BCPC's intrinsic utility and all PICs' utility functions must equal the shadow price λ times the market price p_k. The standard condition $\partial U/\partial x_k = \lambda p_k$ is the special case $\alpha^0_i = 1$. The ratio of conditions for goods k and l yields the Delegated Marginal Rate of Substitution:

(15) $$MRS^D_{kl} = [\alpha^0_i \cdot MU^0_k + \Sigma_j\, \alpha_{ji} \cdot MV_{jk}] / [\alpha^0_i \cdot MU^0_l + \Sigma_j\, \alpha_{ji} \cdot MV_{jl}] = p_k/p_l,$$

where $MU^0_k = \partial U^0_i/\partial x_k$ and $MV_{jk} = \partial V_j/\partial x_k$. The optimal consumption ratio equates this delegated MRS to the price ratio, a generalization of the classical tangency condition.

B. The Influence Elasticity of Demand

- ***Definition 4 (Influence Elasticity of Demand):*** The Influence Elasticity of Demand (IED) of good k with respect to PIC j is:

(16) $$IED_{jk} = (\partial \tilde{x}_k/\partial \omega_{ji}) \cdot (\omega_{ji} / \tilde{x}_k).$$

The IED satisfies the following properties:

1. **Sign:** $IED_{jk} > 0$ if PIC j has stronger preference for good k than BCPC i ($\partial V_j/\partial x_k > \partial U^0_i/\partial x_k$ at the optimum); $IED_{jk} < 0$ otherwise; $IED_{jk} = 0$ if preferences coincide for good k.
2. **Magnitude:** A large positive IED_{jk} indicates high BCPC demand sensitivity to the PIC's influence, the key parameter governing PIC-targeting advertising effectiveness.
3. **Cross-good Effects:** The cross-IED $IED_{jkl} = (\partial\tilde{x}_l/\partial\omega_{ji}) \cdot (\omega_{ji}/\tilde{x}_l)$ captures spillover effects of PIC influence across the consumption bundle.

- ***Proposition 3 (IED Decomposition):*** The IED for good k decomposes into a preference-gap effect and an income-reallocation effect:

(17) $$IED_{jk} = [\partial U^0_i/\partial x_k - \partial V_j/\partial x_k]^{-1} \cdot (\partial^2 DUF_i/\partial x^2_k)^{-1} \cdot (\omega_{ji}/\tilde{x}_k) + R_k,$$

where R_k captures the income reallocation from goods where BCPC demand decreases due to PIC influence. This decomposition mirrors the Slutsky decomposition in structure. See Appendix A for the full proof.

C. Comparative Statics: The Effect of Advertising on Demand

The central policy-relevant result concerns the effect of firm advertising targeted at PICs. Let T_{ji} be advertising expenditure directed at PIC j with the strategic intent of increasing j's influence on BCPC i. From equation (6), $\partial\omega_{ji}/\partial T_{ji} > 0$. The chain rule yields:

(18) $$\partial\tilde{x}_k/\partial T_{ji} = (\partial\tilde{x}_k/\partial\omega_{ji}) \cdot (\partial\omega_{ji}/\partial T_{ji}) > 0 \text{ whenever } IED_{jk} > 0.$$

This formalizes the strategic logic of child-directed advertising for adult-purchased products: advertising that raises a child's preference intensity for good k increases ω_{ji}, which through the DUF increases the BCPC's effective marginal utility of k, shifting demand rightward. The demand effect is achieved without directly modifying the BCPC's intrinsic preferences; it works entirely through the influence pathway. A crucial welfare implication follows: the BCPC may purchase a bundle maximizing DUF_i rather than U^0_i, generating a welfare loss that standard consumer surplus measures cannot capture.

V. IMPLICATIONS FOR CLASSICAL DEMAND THEORY THEOREMS

A. Failure of Slutsky Symmetry

- ***Theorem 1 (Slutsky Symmetry Fails Under PIC Influence):*** In the presence of PIC influence ($\omega > 0$), the Slutsky substitution matrix $\tilde{S}$ associated with the Delegated Demand Function need not be symmetric. Specifically, $\tilde{S}_{kl} \neq \tilde{S}_{lk}$ in general when $\partial\omega_{ji}/\partial p_k \neq 0$.

Proof Sketch. The Slutsky elements for the delegated demand function are:

(19) $$\tilde{S}_{kl} = \partial\tilde{h}_k/\partial p_l = \partial\tilde{x}_k/\partial p_l + \tilde{x}_l \cdot \partial\tilde{x}_k/\partial m_i,$$

where $\tilde{h}_k$ is compensated demand derived from the DUF. Young's theorem guarantees symmetry of the expenditure function $\tilde{e}(p, u) = \min\{p \cdot x : DUF_i(x) \geq u\}$, so $\tilde{S}_{kl} = \tilde{S}_{lk}$ when ω is independent of prices. Asymmetry arises when ω_{ji} is price-dependent, a realistic scenario if, for example, higher prices for good k reduce household consumption of k, reducing the PIC's exposure to the good and thereby the emotional salience E_{ji} of their preference for k. In this case $\partial\omega_{ji}/\partial p_k \neq 0$, and the effective demand system exhibits non-symmetric price responses. ∎

This result aligns with and provides a microeconomic mechanism for Browning and Chiappori's (1998) empirical finding that the Slutsky matrix in collective household models need not be symmetric.

B. Violations of WARP Under Changing Influence Capacity

- ***Theorem 2 (WARP Violations Under Changing Influence Capacity):*** Observed sequences of choices by a BCPC subject to varying PIC influence capacities can violate the Weak Axiom of Revealed Preference (WARP), even when the BCPC's intrinsic preferences U^0_i are perfectly rational and satisfy WARP.

Proof. WARP states: if $\tilde{x}(p_a, m_a) = x_a$ and $p_a \cdot x_b \leq m_a$, then $\tilde{x}(p_b, m_b) \neq x_b$ when $p_b \cdot x_a \leq m_b$. Consider two periods a and b with identical prices and income but different influence capacities $\omega_a \neq \omega_b$ as would occur if a firm increased advertising expenditure T_{ji} between periods. The BCPC chooses $x_a = \tilde{x}(p, m, \omega_a)$ in period a and $x_b = \tilde{x}(p, m, \omega_b)$ in period b. Since x_a is affordable in period b and vice versa (same prices and income), $x_a \neq x_b$ constitutes a WARP violation from the perspective of an observer who does not observe ω_{ji}. Yet no irrationality has occurred; the BCPC rationally maximized DUF_i in both periods, given the different prevailing influence capacities.

This theorem has profound methodological implications. Large-scale investigations of WARP violations including Sippel (1997) and Mattei (2000), who found frequent violations in laboratory settings have typically been interpreted as evidence of irrationality. Our framework offers an alternative: WARP violations in markets with significant PIC influence may reflect changes in influence capacity rather than failures of underlying rationality. This generates a testable hypothesis: WARP violations should be more frequent in product categories with high PIC influence (children's food products, family entertainment, healthcare under caregiver influence) than in categories with low PIC influence.

VI. GRAPHICAL ANALYSIS OF THE DELEGATED CONSUMER OPTIMUM

A. The Preference Displacement Effect

Consider a BCPC with intrinsic preferences U^0_i and a PIC with preferences V_j over two goods: Good 1 (PIC-preferred brand, e.g., Milo) and Good 2 (BCPC-preferred brand, e.g., a generic milk powder). In standard consumer theory, the BCPC would choose bundle A the tangency between the highest attainable U^0_i indifference curve and budget constraint BC. The introduction of PIC influence ($\omega_{ji} > 0$) rotates effective indifference curves toward the PIC's preferred direction, producing a new optimum at bundle B on the same budget constraint. Bundle B allocates more to Good 1 relative to A.

The welfare cost of PIC influence from the perspective of the BCPC's intrinsic preferences is the utility loss measured by the gap between the U^0_i indifference curve through B and the higher U^0_i curve through A. This cost is borne by the BCPC but generated by the firm's PIC-targeting advertising strategy a form of externality standard welfare analysis cannot capture. Figure 1 illustrates.

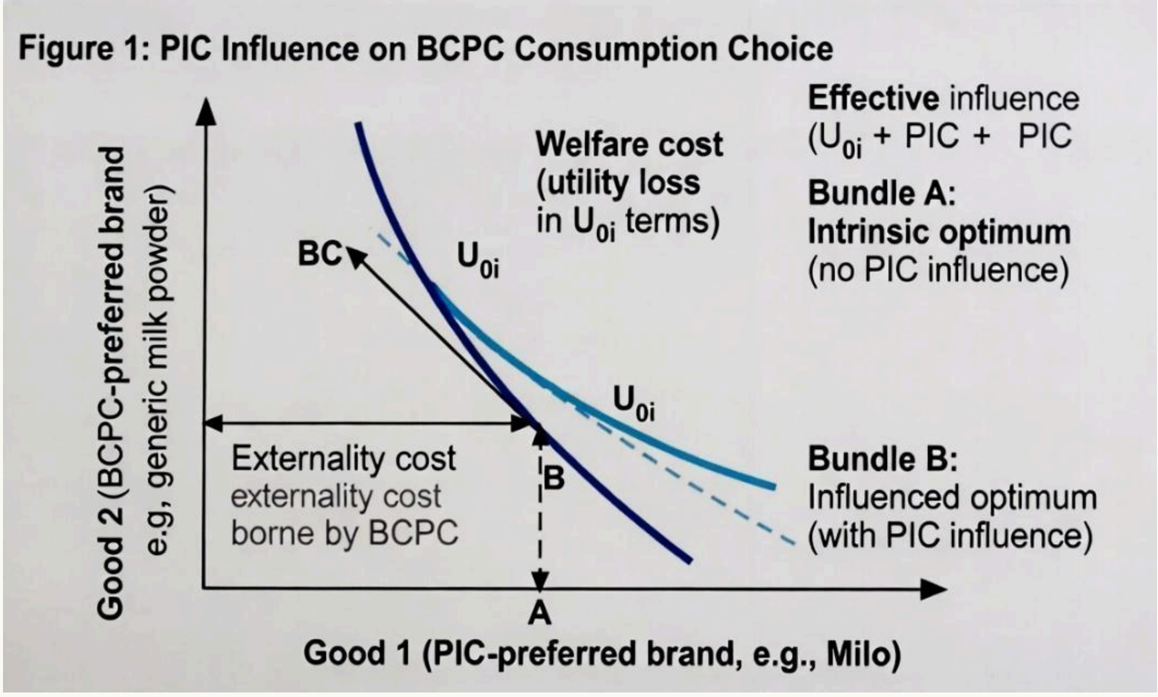

Figure 1 (Google Gemini AI)

B. The Advertising Amplification Effect

Figure 2 illustrates how increasing advertising expenditure T directed at the PIC progressively displaces the BCPC's optimal choice from the intrinsic optimum toward the PIC's bliss point. As ω increases from 0 to 1, the BCPC's chosen bundle traces a path from A ($\omega = 0$, standard optimum) toward F ($\omega = 1$, PIC's bliss point constrained by BC). For the case where the PIC has stronger preferences for Good 1, this demand expansion path is monotone-increasing in Good 1 and monotone-decreasing in Good 2.

The slope of this trajectory in the (x_1, x_2) space equals the ratio $(IED_{j1})/(IED_{j2})$ and determines how efficiently a given increment of advertising expenditure translates into demand shifts. High IED_{j1} combined with negative IED_{j2} implies a steep trajectory toward Good 1, making child-directed advertising highly effective with minimal demand dilution across the bundle.

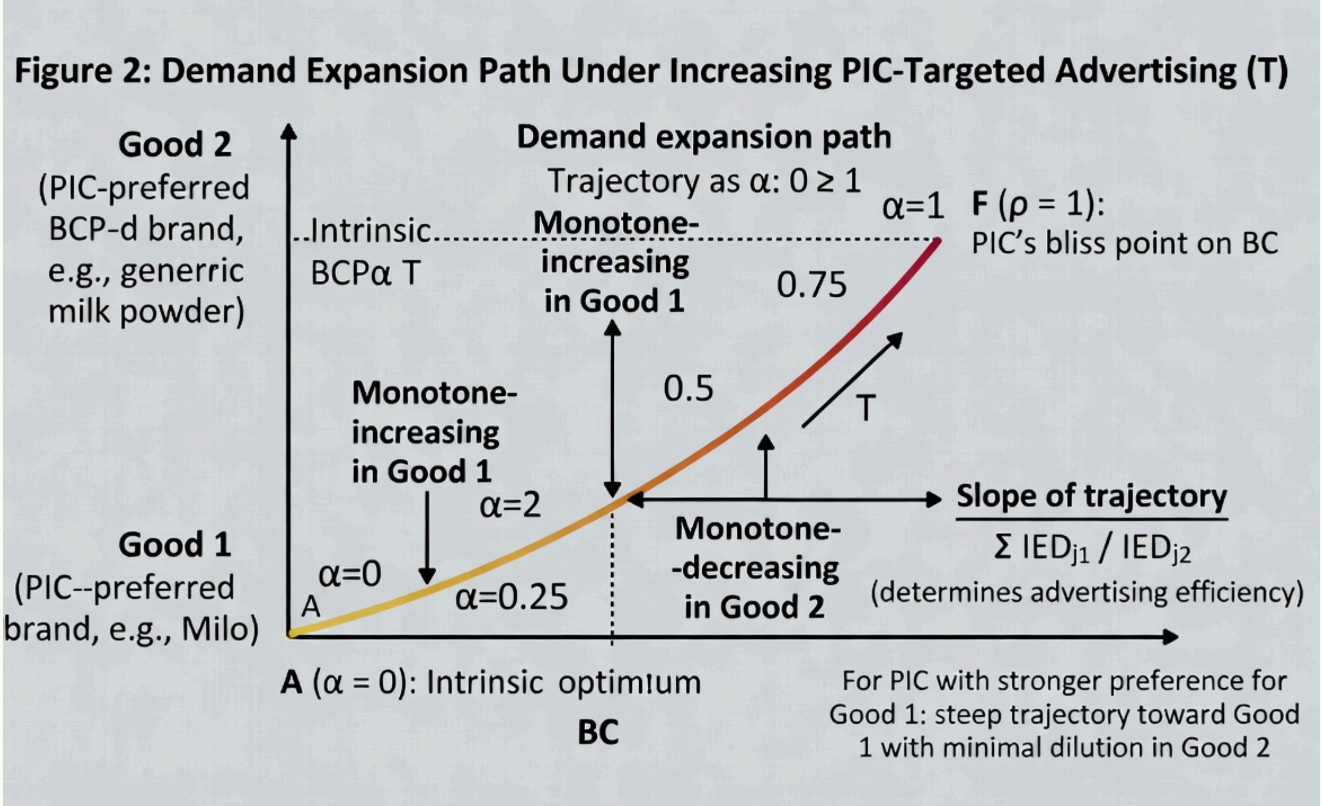

Figure 2 (Google Gemini AI)

VII. EMPIRICAL APPLICATIONS: BEVERAGE MARKETS AND CHILD-DIRECTED ADVERTISING

A. The Milo–Bournvita Case Study

The West African malted beverage market in which Nestlé's Milo and Unilever's Bournvita compete provides one of the cleanest real-world illustrations of PIC-targeting strategy. Both brands allocate significant advertising budgets to child-directed content: animated characters, sports-themed narratives featuring child athletes, and scenes of children exhibiting superhuman energy attributable to consuming the product. The purchasing decision is, in nearly all cases, made by a parent managing a household grocery budget.

Children are PICs: they view advertisements, develop strong brand preferences, express those preferences to parents, and their expressed preferences systematically shift the parental purchasing decision. The firm has engineered the influence chain: firm → child (PIC) → parent (BCPC) → purchase. In terms of our framework, advertising expenditure T_{ji} directed at children raises E_{ji} (emotional salience of the child's expressed preference) and, over repeated exposure, A_{ji} (accumulated preference loading). Both effects increase ω_{ji}. From equation (18), $\partial\tilde{x}\_Milo/\partial T > 0$.

The theoretical framework predicts the following empirically testable patterns: (i) children's stated brand preferences should be highly correlated with household purchasing patterns, controlling for parental intrinsic preferences; (ii) the elasticity of household demand with respect to child-directed advertising expenditure should exceed the elasticity with respect to equivalent parent-directed advertising in high-ω categories; (iii) the BCPC's purchasing decision should be predictable from the PIC's expressed preferences with a lag corresponding to influence transmission time; and (iv) as ω_{ji} increases, BCPC demand for the PIC-preferred brand should rise non-linearly.

B. Econometric Specification

To render our theoretical predictions empirically testable, we propose a household-level demand system. Let h_{it} denote household i's purchase quantity of the focal brand in period t:

$$h_{it} = \beta_0 + \beta_1 \cdot p_{it} + \beta_2 \cdot m_{it} + \beta_3 \cdot \hat{\omega_{jit}} + \beta_4 \cdot (\hat{\omega_{jit}} \cdot p_{it}) + \gamma \cdot Z_{it} + \varepsilon_{it}, \quad (20)$$

where p_{it} is the brand price, m_{it} is household income, $\hat{\omega_{jit}}$ is an empirical proxy for PIC influence capacity (e.g., child television viewing hours of brand advertising, or a survey measure of the child's stated brand preference intensity), Z_{it} is a vector of household controls, and ε_{it} is a disturbance. Our theory predicts $\beta_3 > 0$ (positive influence capacity effect on demand) and $\beta_4 < 0$ (the BCPC is less price-sensitive when PIC influence is high, because children's demand for Milo is not price-contingent and their influence attenuates the parental price response). The ratio β_3/β_1 gives the Influence-Price Substitution Rate: how much influence capacity increase is equivalent in demand terms to a unit price reduction.

The interaction term β_4 is the empirical signature of IED-mediated price insensitivity. A significant negative β_4 implies BCPC demand is more inelastic under high PIC influence with direct implications for firm pricing strategy and antitrust analysis of market power.

VIII. POLICY IMPLICATIONS: MARKETING STRATEGY, REGULATION, AND WELFARE

A. Corporate Marketing Strategy

Our framework yields four strategic propositions that follow directly from the theory.

- **Strategic Proposition 1 (Dual Targeting as Optimization).** Firms should optimize advertising expenditure across two dimensions: BCPC-directed advertising (which modifies U^0_i directly) and PIC-directed advertising (which raises ω_{ji}). When IED is large and intrinsic preference elasticity is low common in categories where BCPC brand preferences are entrenched, the PIC-targeting channel dominates the optimal strategy.
- **Strategic Proposition 2 (Influence Intensity Mapping).** Firms should invest in granular mapping of ω_{ji} across household segments. Influence capacity varies with age of the PIC (child influence capacity tends to peak in the 6–10 age range), household structure, and media consumption patterns. Influence capacity mapping is the dual of standard consumer segmentation: marketers should segment PIC-BCPC dyads by influence capacity parameters, not only BCPCs by demographics.
- **Strategic Proposition 3 (Third-Party Amplification Efficiency).** From equation (6), the advertising multiplier $\partial\omega_{ji}/\partial T_{ji}$ is higher when relational closeness R_{ji} is high. Media channels that reach PICs in emotionally salient household contexts (children watching television with parents, social media content shared within families) generate higher advertising multipliers than channels reaching PICs in isolation.
- **Strategic Proposition 4 (Redefine the Customer).** Firms must formally separate 'who holds purchasing power' (the BCPC) from 'who drives preference formation' (the PIC) in market research, segmentation, and targeting frameworks. Contemporary CRM systems universally define the 'customer' as the purchasing agent. Our theory demonstrates this creates systematic blind spots in high-ω categories.

B. Regulatory and Welfare Implications

Our framework raises concerns about the welfare implications of PIC-targeting advertising strategies, particularly when the PIC is a child. The standard welfare analysis of advertising asks whether advertising provides information enabling better choices. In the PIC-BCPC framework, this question becomes structurally more complex.

Consider a firm directing advertising at a child PIC to increase ω_{ji}, causing a BCPC parent to substitute from a nutritionally superior generic product toward the branded alternative. The advertising provides no new information to the BCPC and the preference modification works through emotional influence rather than information provision. The BCPC consumes a bundle maximizing DUF_i rather than U^0_i a welfare loss in terms of intrinsic preferences, potentially compounded by a nutritional welfare loss for the PIC-child who consumes the final product.

This analysis motivates a new regulatory principle. The welfare standard for advertising regulation should distinguish between advertising targeted at BCPCs (the purchasing agent) and advertising targeted at PICs (influence agents), particularly when PICs are vulnerable. We term this Delegated Preference Distortion (DPD) and propose it as a new category in advertising welfare analysis. Formally:

(21) $$DPD = e(p, U^*) - e(p, U^0_i(\tilde{x})),$$

where $e(p, U^*)$ is the expenditure required to achieve $U^* = U^0_i(x^*(p, m))$ the BCPC's optimum under zero PIC influence and $e(p, U^0_i(\tilde{x}))$ is the expenditure required to achieve the utility the BCPC actually attains at the delegated optimum. $DPD \geq 0$ always, with equality only when $\tilde{x} = x^\star$ (no preference distortion). We propose that regulatory agencies develop DPD as an operational welfare measure alongside standard consumer surplus metrics.

IX. THE HOUSEHOLD AS AN INFLUENCE NETWORK

A. Household Consumption as Aggregated Influence

The preceding analysis treated the PIC-BCPC dyad as a bilateral relationship. In reality, household consumption reflects the influence of multiple PICs on one or more BCPCs, constituting an Influence Network. Let household H consist of n members partitioned into BCPCs $\{i_1, i_2, ..., i_k\}$ and PICs $\{j_1, j_2, ..., j_m\}$. Total household consumption is:

(22) $$X\_H = \Sigma_i\, \tilde{x}_i(p, m_i, \omega_i),$$

where $\omega_i = (\omega_{j1i}, \omega_{j2i}, ..., \omega_{jmi})$ is the vector of influence capacities on BCPC i. The household demand system is thus a function of prices, incomes, and the entire influence network structure generating predictions about how household composition affects consumption patterns that are entirely absent from standard household demand models, including the Almost Ideal Demand System (Deaton and Muellbauer 1980) and QUAIDS (Banks, Blundell, and Lewbel 1997).

B. Spousal Influence and Joint Household Consumption

A particularly important special case is the non-earning spouse in a household where one partner controls income. In many cultural contexts, one partner holds formal purchasing authority (BCPC) while the other influences preferences and consumption decisions without formal income of their own, a textbook PIC-BCPC structure with ω_{ji} potentially very high.

The standard unitary household model (Samuelson 1956; Becker 1965) treats the household as a single utility-maximizing agent, collapsing the PIC-BCPC distinction. The collective model (Browning and Chiappori 1998; Apps and Rees 1997) recognizes multiple agents but treats them symmetrically as Pareto bargainers. Our framework offers a third perspective: the non-earning partner is a PIC whose influence capacity ω_{ji} is determined by relationship quality, cultural norms, and the social recognition of non-monetary contributions to household welfare. This has important implications for policy: programs targeting resources at women in male-income-controlled households must treat ω_{ji} as a crucial mediating variable. The standard finding that women's income disproportionately benefits child nutrition (Thomas 1990) may partly reflect high ω_{ji} in households where women are de facto PICs.

X. A REVISED AXIOMATIC FOUNDATION: STRATIFIED CONSUMER THEORY

The preceding analysis motivates a formal revision of the axiomatic foundations of consumer theory. We propose replacing Assumptions C1–C4 with the following:

- ***Axiom A1 (Stratified Agency):*** Economic agents in the consumer domain are classified into BCPCs (budget-holding preference agents) and PICs (non-budget preference influencers). Both types form well-defined preferences, but only BCPCs face binding budget constraints and hold purchase authority.
- ***Axiom A2 (Endogenous Preference Formation):*** A BCPC's effective preference ordering is endogenous to the influence network in which they are embedded. The effective preference ordering is a weighted composition of the BCPC's intrinsic preferences and the preferences of PICs in their network, with weights determined by the influence capacity function $\omega(R, E, A, T)$.
- ***Axiom A3 (Stratified Preference Authority):*** Preference authority and budgetary authority are held by the same agent (the BCPC) but are not identical in magnitude. A BCPC with influence capacity vector ω faces intrinsic preference authority $1 - \Sigma_j \omega_{ji}$, approaching zero as PIC influence approaches unity.
- ***Axiom A4 (Multi-Lateral Market Structure):*** Relevant market interactions involve at minimum a trilateral structure: the firm, the PIC, and the BCPC. Firm strategy encompasses both BCPC-directed channels (direct modification of U^0_i) and PIC-directed channels (indirect modification via raising ω_{ji}). A complete theory of market equilibrium must specify strategies across both channels.

These four axioms, together with standard regularity conditions on utility functions, constitute what we term Stratified Consumer Theory (SCT). SCT nests the standard model (as the special case $\omega_{ji} = 0 \ \forall j$) and the collective household model (as the special case where all PICs are also BCPCs with positive income), while encompassing a substantially broader class of consumer interactions.

XI. FUTURE RESEARCH AGENDA

A. Empirical Measurement of Influence Capacity

Empirical implementation requires reliable measurement of ω_{ji}. We identify three approaches. First, survey instruments that directly elicit BCPCs' assessment of PIC influence on recent purchasing decisions. Second, behavioral experiments using dictator-game variants in which the BCPC makes purchasing decisions while the PIC's preferences are revealed with varying intensity allowing identification of ω_{ji} from choice behavior. Third, observational panel data from household scanner datasets combined with child advertising exposure measures (set-top box viewing data) to estimate the reduced-form equation (20) and recover structural IED parameters.

B. Dynamic Influence Capacity and Habit Formation

The present paper treats ω_{ji} as a static parameter manipulable through advertising. A richer dynamic extension would model influence capacity accumulation as a state variable, with PIC-directed advertising in period t affecting the transition of ω_{ji} to period t + 1. This generates a brand loyalty dynamics model in which long-run effects of child-directed advertising substantially exceed immediate demand effects consistent with the long-run brand equity effects documented in the marketing literature.

C. Strategic Interaction in Oligopoly with PIC Channels

In a duopoly where both firms can direct advertising at the child PIC, equilibrium advertising strategies form a game over influence capacities. The firm-level first-order conditions for advertising expenditure must account for the influence-capacity externality: advertising by Firm A that raises child preference intensity for Brand A simultaneously reduces influence capacity available for Brand B. This generates an influence arms race with potentially inefficient equilibrium advertising levels, a new mechanism for advertising market failure.

XII. CONCLUSION

This paper has developed a comprehensive theoretical framework for understanding consumer choice when the agent holding purchasing power is systematically influenced by a second agent with no purchasing power but whose preferences are nonetheless economically consequential. We have defined the Proxy-Influence Consumer (PIC), introduced the Delegated Utility Function as the formal representation of influence-modified preferences, derived new equilibrium conditions, the Influence Elasticity of Demand, and proved that Slutsky symmetry and WARP fail under conditions of active PIC influence.

The theory addresses a real and growing empirical phenomenon. In modern economies, the decoupling of preference formation from purchasing power has become structurally significant: children influencing family consumption; non-earning partners shaping household expenditure; social media influencers modifying followers' preferences; patients in healthcare settings whose drug demand is mediated by caregivers. Standard consumer theory, built on the axiom that the preferrer is the payer, generates systematically biased predictions in all these contexts.

The practical implications are sharp. Firms in FMCG markets particularly beverage, food, entertainment, and personal care categories are already implicitly exploiting the PIC-BCPC architecture. The Milo and Bournvita case illustrates a market strategy that is rational, effective, and welfare-diminishing from the BCPC's intrinsic preference standpoint, yet entirely invisible to standard demand models. Our framework provides the theoretical language to understand these strategies, measure their effectiveness via the IED, and evaluate their welfare consequences via Delegated Preference Distortion.

For economic theory, the framework calls for a revision of the axioms of consumer theory. The four classical assumptions we identified: Preference-Expenditure Unity, Preference Exogeneity, Unitary Preference Authority, and Bilateral Market Structure fail systematically, predictably, and in economically important markets. The four replacement axioms of Stratified Consumer Theory offer a more empirically accurate foundation for microeconomics in the twenty-first century.

The consumer who matters economically need not be the consumer who pays.

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FIGURE CAPTIONS

Figure 1. Preference Displacement Effect. *Note:* Point A denotes the BCPC's standard optimum (maximizing U^0_i); Point B denotes the Delegated Optimum (maximizing DUF_i with $\omega > 0$). The budget constraint BC is unchanged. The displacement A→B reflects the preference-weighted shift toward Good 1 (the PIC-preferred brand) driven by PIC influence. The DUF indifference curve through B is less steep (lower intrinsic MRS) but the PIC-weighted MRS equals the price ratio at B. The shaded area represents the welfare loss DPD measured in expenditure units.

Figure 2. Advertising Amplification Effect: Demand Trajectory as ω Increases from 0 to 1. *Note:* Left panel plots quantity demanded of Good 1 (x_1) and Good 2 (x_2) as functions of $\omega \in [0,1]$, holding prices and income fixed. As ω increases from 0 (standard model) to 1 (complete PIC delegation), x_1 increases monotonically and x_2 decreases monotonically for the case $IED_{j1} > 0 > IED_{j2}$. The right panel shows the corresponding trajectory in the (x_1, x_2) commodity space, from the

BCPC intrinsic optimum A ($\omega = 0$) toward the PIC's budget-constrained bliss point F ($\omega = 1$). The slope of this trajectory equals $(IED_{j1})/(IED_{j2})$.

APPENDIX A: MATHEMATICAL PROOFS

A. Proof of Proposition 3 (IED Decomposition)

We prove the Influence Elasticity of Demand decomposition (equation 17) by total differentiation of the first-order conditions with respect to ω_{ji}.

From the first-order conditions (14), at an interior optimum:

(A1) $\alpha^0_i \cdot \partial U^0_i/\partial x_k + \Sigma_j\, \alpha_{ji} \cdot \partial V_j/\partial x_k = \lambda \cdot p_k \quad \forall k,$

(A2) $p \cdot x = m_i.$

Differentiating (A1)–(A2) totally with respect to α_{ji}:

(A3) $\Sigma_l\, [\partial^2 DUF_i/\partial x_k \partial x_l] \cdot (\partial x_l/\partial \alpha_{ji}) - p_k \cdot (\partial \lambda/\partial \alpha_{ji}) = -(\partial V_j/\partial x_k - \partial U^0_i/\partial x_k),$

(A4) $\Sigma_k\, p_k \cdot (\partial x_k/\partial \alpha_{ji}) = 0.$

Let $H = [\partial^2 DUF_i/\partial x_k \partial x_l]$ be the bordered Hessian of the DUF, p be the price vector, and $\Delta_j = [\partial V_j/\partial x_k - \partial U^0_i/\partial x_k]_k$ be the preference-gap vector. In matrix form:

(A5) $[\ H \quad -p\ ]\,[\partial x/\partial \alpha_{ji}] \quad [-\Delta_j][-p^T \quad 0\ ]\,[\partial \lambda/\partial \alpha_{ji}] = [\ 0\ ].$

The solution by Cramer's rule yields the preference-gap effect (the first term in equation 17). The income-reallocation effect R_k arises because constraint (A4) forces any increase in x_k to be offset by decreases in other goods, the standard budget-balance spillover. ∎

B. Lemma A1: Sufficient Condition for Positive IED

Lemma A1. $IED_{jk} > 0$ if and only if $\partial V_j/\partial x_k > \partial U^0_i/\partial x_k$ at the optimum that is, the PIC has a strictly higher marginal utility for good k than the BCPC at the consumption bundle solving the extended consumer problem.

Proof. From equation (17), the sign of IED_{jk} is determined by the sign of the preference-gap term $[\partial U^0_i/\partial x_k - \partial V_j/\partial x_k]^{-1} \cdot (\partial^2 DUF_i/\partial x^2_k)^{-1} \cdot (\omega_{ji}/\tilde{x}_k)$. Under strict quasi-concavity, $\partial^2 DUF_i/\partial x^2_k < 0$ at the optimum. Since $\omega_{ji} > 0$ and $\tilde{x}_k > 0$ at an interior solution, the sign of IED_{jk} equals the sign of $-[\partial U^0_i/\partial x_k - \partial V_j/\partial x_k]$, which is positive if and only if $\partial V_j/\partial x_k > \partial U^0_i/\partial x_k$. ∎

The IED is positive precisely when the PIC 'wants more' of good k than the BCPC would intrinsically choose. In the child-beverage case: the child's marginal utility from the branded beverage (Milo) exceeds the parent's intrinsic marginal utility (the parent would prefer the cheaper generic), so PIC influence has positive IED shifting parental demand toward the brand.

C. Proof of Theorem 2 (WARP Violations)

We provide a constructive proof by example that rationalizes apparent WARP violations via changing influence capacity.

Let $n = 2$, $p = (1, 1)$, $m_i = 10$. In period a, $\omega_a = 0.1$ (low PIC influence), so $DUF_{ia} \approx U^0_i$ and the BCPC chooses $x_a = (3, 7)$, a bundle tilted toward Good 2. In period b, $\omega_b = 0.9$ (high PIC influence following an advertising campaign), so $DUF_{ib} \approx V_j$ and the BCPC chooses $x_b = (7, 3)$, a bundle tilted toward Good 1. Since $p_a \cdot x_b = 10 \leq m_i = 10$ and $p_b \cdot x_a = 10 \leq m_i = 10$, x_a was affordable in period b and x_b was affordable in period a. Yet $x_a \neq x_b$ a WARP violation in the data, rationalized entirely by the change in ω. No irrationality has occurred; the BCPC has rationally maximized DUF_i in each period. ∎